Spatial Equilibrium: Roback 1982 and Moretti 2010

Nathan Schiff
Shanghai University of Finance and Economics

Graduate Urban Economics, Week 5
March 25, 2024

How do we price negative effect of pollution?

Most cost-benefit analyses of “bads”, like pollution, need some measure for negative effect of pollution

In order to evaluate policy we usually calculate a marginal willingness to pay; this can be compared to cost of implementing some pollution reduction policy

So, if you wanted to quantify the negative effect of pollution on Beijing, how would you do it?

What would you try to measure? How would you report your results?

What method would you use to identify this effect? What data would you use?

Inferring MWTP from Housing Prices

In AMM monocentric city model, we already saw that value of being close to center can be inferred from housing prices

We can use this idea generally. Say **a** is distance from a pollution source, consumers consume one unit of housing, and spend rest of money on a numeraire **x**.

Housing price will adjust with **a** to compensate consumers for being close to pollution, then:

$U(x; a) = k$ and $w - p(a) = x$

Totally differentiating: $\frac{dU}{da} = \frac{\partial U}{\partial x}\frac{dx}{da} + \frac{\partial U}{\partial a} = \frac{\partial U}{\partial x}\frac{-dp(a)}{da} + \frac{\partial U}{\partial a} = 0$

MWTP for distance from polluter: $\frac{\partial U}{\partial a} / \frac{\partial U}{\partial x} = \frac{dp}{da}$

Spatial Equilibrium Framework

Spatial equilibrium models describe how people migrate in response to changes in location characteristics

We can invert this relationship to try and back out the value (or cost) of location specific factors from migration patterns

In the spatial equilibrium framework workers have utility over tradable goods, non-tradable goods (ex: housing), and the location specific factors ("amenities")

Since utility must be equal across locations, wages and housing prices adjust to make workers indifferent

Therefore we can infer the value of amenities from wages and housing prices

Wages, Rents, and the Quality of Life, JPE 1982

Very famous paper from Roback's 1980 thesis, over 4000 citations

Not only important to Urban Economics literature but also quite important to Labor (local labor markets–see references on last slide), Environmental Economics (pricing of environmental amenities), Trade (markets and industry concentration), and Development (migration)

Many theoretical extensions to consider heterogeneous agents, tax policy, agglomeration and congestion

Recent work uses framework in quite rigorous empirical estimation of quality of life and city wage differentials (see Albouy, "Are Big Cities Bad Places to Live? Estimating Quality of Life across Metropolitan Areas.")

Jennifer Roback, “Wages, Rents, and the Quality of Life,” JPE, 1982

Microeconomics Theory Refresher

Main tools of Roback model are indirect utility function and unit cost function; she uses Shepard's Lemma and Roy's Identity to derive some results

Shepard's Lemma: derivative of cost function $C(W, y)$ with respect to price of an input, w_i, is conditional factor demand for that input $z_i(w_i, y)$:

$$z_i(w_i, y) = \frac{\partial C(W, y)}{\partial w_i} \qquad \text{(Shepard's Lemma)}$$

Roy's Identity: negative ratio of derivatives of indirect utility function w.r.t. price of a good p_i and income m is Marshallian demand for that good

$$x_i(p_i, m) = -\frac{\partial V(P, m)}{\partial p_i} \Big/ \frac{\partial V(P, m)}{\partial m} \qquad \text{(Roy's Identity)}$$

Workers

Identical workers with cost-less migration, each supplies one unit of labor

Different cities have different *exogenous* amenities (ex: warm climate, natural beauty, clean air), denoted s

Worker utility is function of s, consumption of composite commodity X (numeraire, paid with wage w), and consumption of land l^c (rented at r)

$$\max_{x,l^c} U(x, l^c; s) \text{ s.t. } x + r * l^c = w + I \tag{1}$$

Free migration ensures spatial equilibrium condition of equal utility:

$$V(w, r; s) = k \tag{2}$$

Assume amenity increases utility: $V_s = \partial V / \partial s > 0$

Firms

Firms produce X with CRS production function $X = f(L^p, N; s)$, where N is number of workers and L^p is land used in production

In equilibrium the unit cost (CRS) must equal price of X (assumed to be 1); if not firms could relocate to more profitable cities, which would increase factor prices (land, labor) in those cities until equilibrium is reached

$$C(w, r; s) = 1 \tag{3}$$

Given a unit cost function and Shepard's Lemma we have that $C_w = N(w, r, X; s)/X$ and $C_r = L^p(w, r, X; s)/X$

The amenity may be unproductive $C_s > 0$ or productive $C_s < 0$ (typo in paper says unproductive is $C_s < 0$)

An amenity that is positive for consumers (clean air) may be unproductive for firms (clean air regulation may require expensive non-polluting technology)

Equilibrium

In equilibrium wages and rents adjust so that workers and firms are indifferent across locations:

$$V(w, r; s) = k \quad (2)$$

$$C(w, r; s) = 1 \quad (3)$$

$$L^p + N * l^c = L \quad \text{(Land Constraint)}$$

Can use equations 2) and 3) to determine wages w and rent r

Easiest way is to graph isocost curves ($C = 1$) and indifference curves ($V = k$) in wage w and rent r space

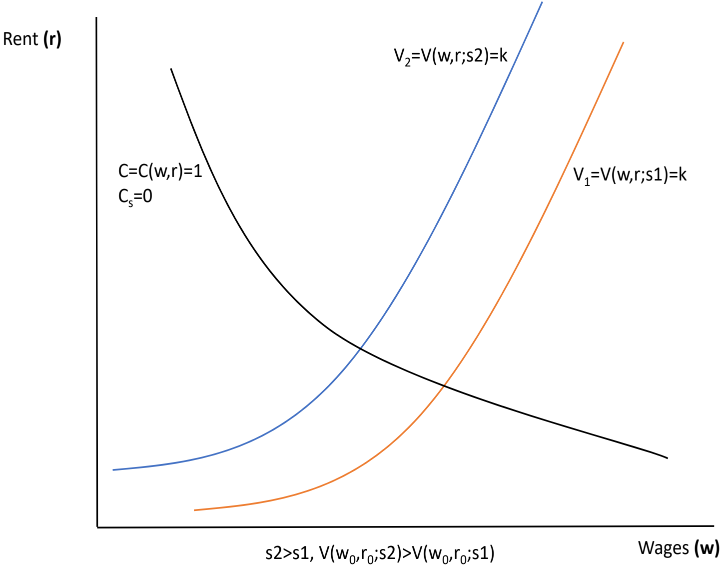
Can vary amount of amenity s to see effect on curves and solve for equilibrium levels of w and r

Benchmark Cases

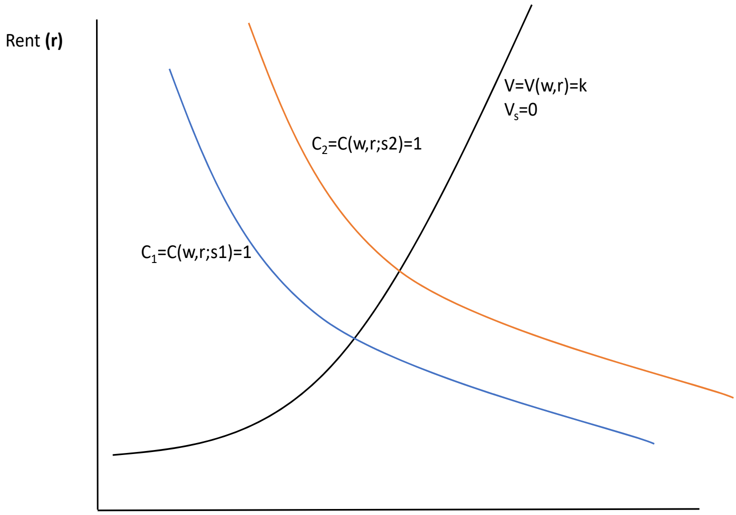
In David Card's lecture notes he considers some simple cases to build up the intuition before considering Roback's Figure 1

- Say that the amenity has no effect on production ($C_s = 0$) and one city has nicer weather than another, $s_2 > s_1$, $V_s > 0$. How will wages and rent compare across the two cities? Will both wages and rent adjust or is one enough?
- What if the amenity has no effect on utility but does lower the cost of production: $s_2 > s_1$, $C_s < 0$, $V_s = 0$? For example, one city has access to cheap hydroelectricity (from an ugly river).
- Consider again a consumer amenity but for the case where not only does the amenity have no effect on production, but *also* land is not even used in production.

Consumer Amenity Only

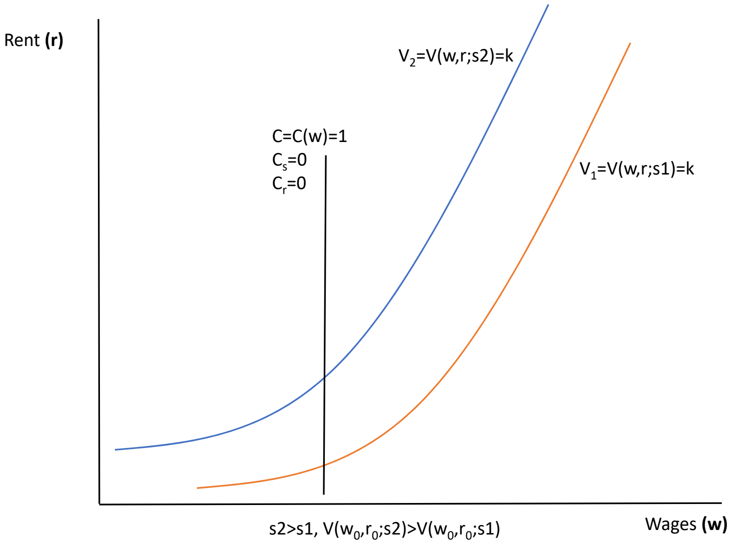


Producer Amenity Only



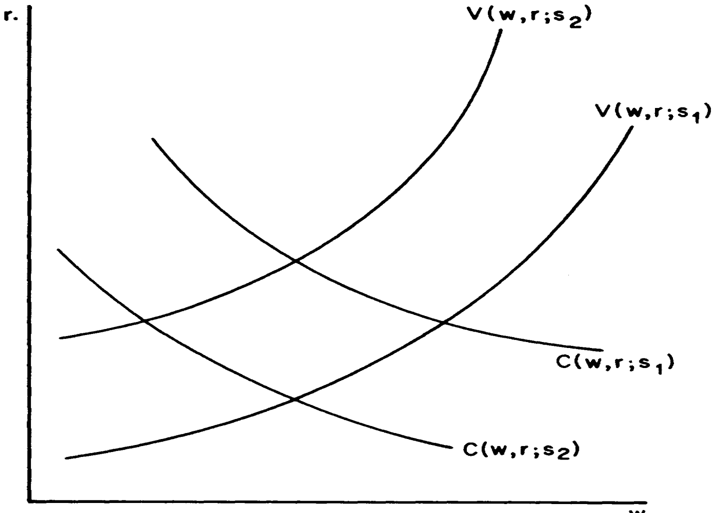
s2>s1, $C(w_0,r_0;s1)>C(w_0,r_0;s2)$, or $C_s<0$

Consumer Amenity, No Land in Production



Consumer Amenity, Producer Disamenity

What if s is valued by consumer but increases cost of producer?



Equilibrium Wage and Rent

If s is valued by consumers but a disamenity for producers then wages are lower in s_2 city but rent may be higher or lower

Similarly, if s is valued by producers but a disamenity for consumers (ex: low pollution standards) then rent will be higher in low s city but wages are uncertain

Differentiate both equilibrium conditions, $V(w, r; s) = k$ and $C(w, r; s) = 1$ w.r.t. s:

$$\frac{dw}{ds} = \frac{-V_s * C_r + C_s * V_r}{V_w * C_r - V_r * C_w} \quad (4)$$
$$\frac{dr}{ds} = \frac{-V_w * C_s + V_s * C_w}{V_w * C_r - V_r * C_w}$$

Denominator is always positive (next slide) so if $V_s > 0$ and $C_s > 0$ then $dw/ds < 0$ but $dr/ds <> 0$

Using Roy's Identity for Consumer Land Demand

$$V(r,w;s) = \max_{x,l^c} U(x,l^c;s) \text{ s.t. } x + r * l^c - w - I = 0 \quad (1)$$

Let λ be marginal utility of additional income from Lagrangian:

$$\frac{\partial V}{\partial w} = V_w = \lambda$$
$$\frac{\partial V}{\partial r} = V_r = -\lambda * l^c$$

This gives us $V_r = -V_w * l^c$, which we also know from Roy's identity (ratio of derivative of indirect utility w.r.t. price and w.r.t. wealth is equal to Marshallian demand)

Then since $C_r = L^p/X$ and $C_w = N/X$, we know that
$V_w * C_r - V_r * C_w = V_w * l^p/X - (-V_w * l^c) * (N/X) = V_w * L/X > 0$

Marginal Willingness to Pay

Using the equilibrium condition we can infer the value of the amenity from changes in wages and rents

$$\Omega(s) = V(w(s), r(s), s) = k$$
$$\frac{d\Omega(s)}{ds} = \frac{\partial V}{\partial w}\frac{dw}{ds} + \frac{\partial V}{\partial r}\frac{dr}{ds} + \frac{\partial V}{\partial s} = 0$$

Using Roy's identity we get:

$$Vs = -V_w * \frac{dw}{ds} + V_w * l^c * \frac{dr}{ds}$$
$$p_s^* \equiv \frac{V_s}{V_w} = l^c * \frac{dr}{ds} - \frac{dw}{ds} \qquad (5a)$$

This p_s^* is the marginal willingess to pay for an additional unit of the amenity s

Marginal Productivity of Amenity

Similarly, we can derive the productivity effect (change in firm cost) of an amenity

$$\Omega(s) = C(w(s), r(s), s) = 1$$
$$\frac{d\Omega(s)}{ds} = \frac{\partial C}{\partial w}\frac{dw}{ds} + \frac{\partial C}{\partial r}\frac{dr}{ds} + \frac{\partial C}{\partial s} = 0$$

Using Shepard's Lemma (derivative of cost functions is input factor demand):

$$C_s = -C_w * \frac{dw}{ds} + -C_r * \frac{dr}{ds}$$
$$C_s = -\left(\frac{N()}{X}\frac{dw}{ds} + \frac{l^p()}{X}\frac{dr}{ds}\right) \tag{5b}$$

Thus C_s is marginal effect of the amenity on cost

Total Marginal Effect of an Amenity

If an amenity s affects consumers through utility *and* firms through productivity, what is the total marginal effect?

Effect on N consumers is aggregate willingness to pay:
$N * p_s^* = N * l^c * \frac{dr}{ds} - N * \frac{dw}{ds}$

Effect on firms is total change in production: $-C_s * X = X * (-C_w * \frac{dw}{ds} + -C_r * \frac{dr}{ds})$

But since $C_s = -\left(\frac{N()}{X}\frac{dw}{ds} + \frac{l^p()}{X}\frac{dr}{ds}\right)$ and $L = L^p + N * l^c$, we have:

Total change=$N * p_s^* - C_s * X = L * \frac{dr}{ds}$

Thus the wage effects exactly cancel each other out (worker gain in wages is a firm loss) and total effect (of marginal change in s) is simply the change in value of land

Applications and extensions of the Roback mdoel

Quality of Life and Valuing Amenities

$$p_s^* \equiv \frac{V_s}{V_w} = I^c * \frac{dr}{ds} - \frac{dw}{ds} \tag{5}$$

Albouy (2012) takes the total differential of the spatial equilibrium equation and log-linearizes around national averages of wages and prices; this gives him an index of quality of life that does not require choosing amenities (basically a residual from wages and prices for traded and non-traded goods)

The application of Roback (and Glaeser Gottlieb) is to value specific amenities (denote with z) by estimating dr/dz and dw/dz using regressions of rents on z and wages on z across cities with different amounts of z

Roback then multiplies the calculated weight p_z^* times the amount of the attribute z in a city, for all attributes $z \in Z$, to get a measure of quality of life

Example Valuation

Say z is a bad amenity, such as crime or pollution; we want to know how people value a reduction in this bad

Roback re-writes eq5 as budget shares (easier estimation)

$$p_z^* \equiv \frac{V_z}{V_w} = l^c * \frac{dr}{dz} - \frac{dw}{dz} = w \left[\frac{l^c * r}{w} * \frac{dr}{dz} * \frac{1}{r} - \frac{dw}{dz} * \frac{1}{w} \right]$$

$$\frac{p_z^*}{w} = k_l \frac{d \log r}{dz} - \frac{d \log w}{dz} = k_l * \gamma_r - \gamma_w$$

In the above eq. k_l is the share of budget spent on land

Then we take an estimate of k_l, run regressions for γ's, and plug back into eq 5:

$$\log w_{ic} = x_i \beta + \gamma_w * z_c + \epsilon_{ic}$$

$$\log r_c = \gamma_r * z_c + \mu_c$$

"Valuing Air Quality in Chinese Cities," Huang and Lanz, 2018

Huang and Lanz estimate willingness to pay for air quality with data on 288 Chinese cities in 2011 (student presentation?)

Use Roback framework to write marginal willingness to pay for pollution as:

$$MWTP_Q = -\frac{dw}{dQ} + H\frac{dr}{dQ} \tag{5}$$

$$Wages_i = \alpha_0 + \alpha_1 AirPollution_i + \alpha_2 HousePrices_i + \alpha_3 X_i^{wages} + \epsilon_i^{wages} \tag{6}$$

$$HousePrices_i = \beta_0 + \beta_1 AirPollution_i + \beta_2 Wages_i + \beta_3 X_i^{HousePrices} + \epsilon_i^{HousePrices} \tag{7}$$

Find that "willingness to pay for a unit reduction in PM_{10} is CNY 261, with a significant share reflected in labor market outcomes"

Interestingly, Chay and Greenstone (JPE 2005) find that in US entire effect of air pollution is reflected in house prices, nothing in wages

Roback's Extensions

Roback then extends the basic model by introducing a non-tradable goods (housing) sector

This sector also competes for land use; incorporating this sector allows author to derive effect of change in s on utility as function of house price changes *and* wages

Glaeser and Gottlieb extend this set-up even further and look more deeply at empirical implications

Moretti 2010; Hsieh and Moretti 2019

Moretti points out that basic version of Roback model assumes i) workers are perfectly mobile (labor supply is infinitely elastic) and ii) housing supply elasticity is fairly limited

These two assumptions imply that an amenity or productivity change is fully capitalized into housing prices, thus only landowners benefit from a positive change

Moretti (2010) uses a simple and intuitive model that allows for limited worker mobility and elastic housing. Productivity and amenity shocks are not fully capitalized into housing prices; for example, workers in a city can then benefit from a productivity increase

A famous paper by Hsieh and Moretti (AEJ 2019) study the “misallocation” that occurs due to housing supply restrictions

When cities restrict housing supply (zoning, NIMBY), productivity shocks lead to large increases in housing prices and nominal wages, not in-migration

Example of Spatial Equilibrium Model: Moretti, Handbook of Labor Economics, 2010

Moretti's Two City Model, Dropping Capital

Moretti's handbook article offers a simple way to implement the Roback model with linear utility and 2 cities

Two important differences:

1. consumers are heterogeneous with idiosyncratic preferences for each city
2. firms do not use land in production

Since firms don't use land and the production function is Cobb-Douglas, this case would correspond to the vertical isocost curve (in rent and wage space) of Roback's model

To make the analysis more interesting, ***I will exclude capital from the production function*** so that output is a concave function of labor supply (population)

This small change makes the wage in each city dependent upon the population, and thus firms will care about indirectly about rent levels (since these affect population)

Two City Model: Each City Affects the Other

In Roback (1982), a single city is assumed to be tiny relative to the national population, thus changes in one city have no effect on other cities

In Moretti's model, if population flows into *a* then it must flow out of *b*, thus cities affect each other through migration

We can do partial equilibrium analysis similar to the Roback model by analyzing one city, allowing for population changes in that city, but assuming that characteristics of the other city are fixed

This will allow us to draw partial equilibrium plots of rents vs wages, similar to the Roback model figures

However, we can also show the general equilibrium effect, in which migration from one city to the other affects both cities

Because consumers have individual preferences for each city, not all changes are fully offset by migration (some people stay), and thus utility can change (more next class on "Place-based Policies")

Consumers

There are two cities (a, b) with N_c consumers in each city; $N_a + N_b = N$

Indirect utility of consumer i in city c, with city amenity A_c and wage w_c:

$$U_{ic} = w_c - r_c + A_c + \epsilon_{ic} \tag{1}$$

Each consumer consumes a single unit of housing and pays r_c. The idiosyncratic ϵ_{ic} term is distributed uniformly such that:

$$\epsilon_{ia} - \epsilon_{ib} \sim U[-s, s] \tag{2}$$

The s term determines strength of preferences for a location (and labor mobility)

When s is large consumers have greater taste for a or b and are less likely to move in response to a marginal change in attractiveness of a city

Migration, Population in each city

In this model, only the *marginal* worker is indifferent between the two cities

A worker chooses a if $U_{ia} > U_{ib}$, which implies that $\epsilon_{ia} - \epsilon_{ib} > (w_b - r_b) - (w_a - r_a) + (A_b - A_a)$

Since $\epsilon_{ia} - \epsilon_{ib} \sim U[-s, s]$, we know:

$$Pr(\epsilon_{ia} - \epsilon_{ib} < x) = \frac{1}{2s}\int_{r=-s}^{x} dr = \frac{x+s}{2s} = \frac{N_b}{N}$$

Then, $Pr(\epsilon_{ia} - \epsilon_{ib} > x) = \frac{s-x}{2s} = \frac{N_a}{N}$. If $x = (w_b - r_b) - (w_a - r_a) + (A_b - A_a)$:

$$\frac{N_a}{N} = \frac{1}{2} + \frac{w_a - r_a}{2s} - \frac{w_b - r_b}{2s} + \frac{A_a - A_b}{2s} \tag{1}$$

Firms

Production function for firms in city c is a concave function of labor (population) N_c:

$$ln(y_c) = X_c + hN_c + (1-h)K_c \quad (4.alt)$$

Notice that land/housing doesn't enter the production function, an important difference with the Roback model

However, we will see that firms are affected by rents through changes in population, which affects marginal productivity of workers

Firms are price-takers and labor is paid marginal product:

$$w_c = X_c - (1-h)N_c + ln(h) \quad \text{(5.alt: Labor Demand)}$$

Closing Model

From migration equation we can write inverse local supply of labor and the demand for housing:

$$w_b = w_a + (r_b - r_a) + (A_a - A_b) + s\frac{(N_b - N_a)}{N} \qquad \text{(3: Labor Supply)}$$

$$r_b = (w_b - w_a) + r_a + (A_b - A_a) - s\frac{N_b - N_a}{N} \qquad \text{(6: Housing Demand)}$$

Moretti assumes that housing supply is equal to the population of workers, who each consume one unit

Inverse supply of housing, with k_c determining inverse elasticity:

$$r_c = z + k_c N_c \qquad \text{(7: Housing Supply)}$$

Equilibrium

In equilibrium, labor supply equals labor demand (eq 3 = eq 5.alt) and housing supply equals housing demand (eq 6 = eq 7)

$$w_b = X_b - (1-h)N_b + ln(h) \qquad \text{(5.alt: Labor Demand)}$$

$$w_b = w_a + (r_b - r_a) + (A_a - A_b) + s\frac{(N_b - N_a)}{N} \qquad \text{(3: Labor Supply)}$$

$$r_b = (w_b - w_a) + r_a + (A_b - A_a) - s\frac{N_b - N_a}{N} \qquad \text{(6: Housing Demand)}$$

$$r_b = z + k_b N_b \qquad \text{(7: Housing Supply)}$$

Equilibrium with no migration: $N_c = \bar{N}_c$

If the population of a city is fixed then labor supply and housing demand are both perfectly inelastic (equal to $\bar{N}_c$)

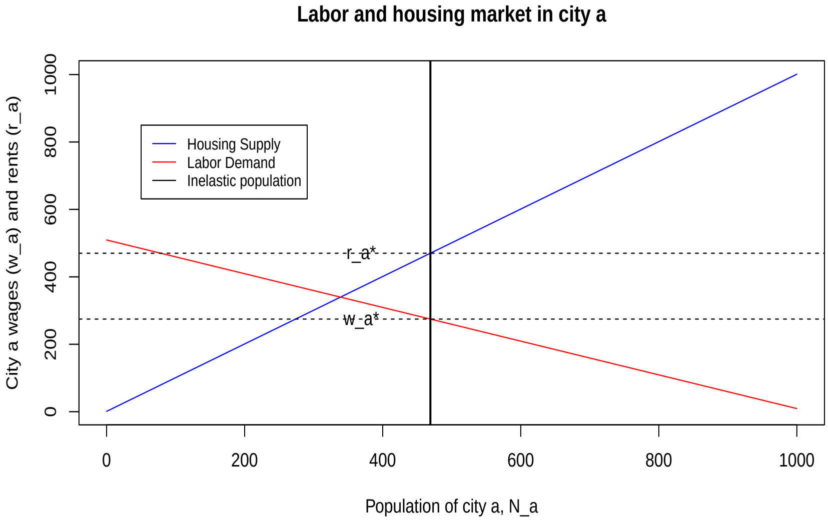
Wages are determined from marginal productivity: $w_c = X_b - (1-h)\bar{N}_c + ln(h)$

Rents come directly from housing supply: $r_c = z + k_c\bar{N}_c$

If consumer amenities increase there is no population inflow, thus consumer utility increases with no other effects (rent and wages are constant)

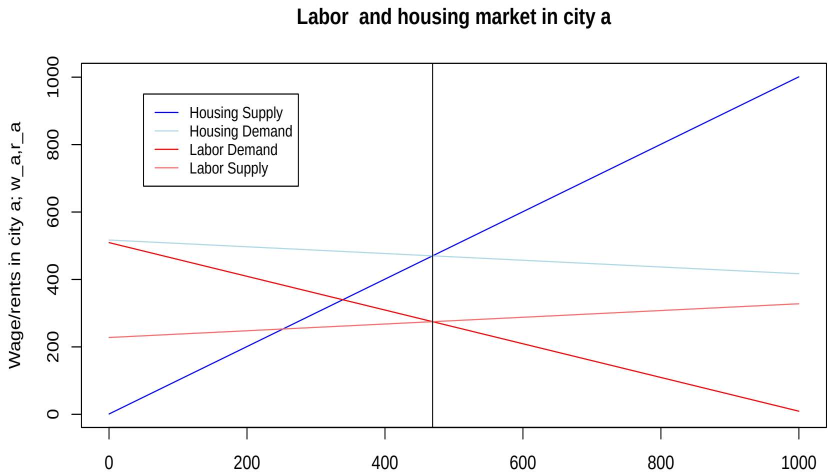
Similarly, if productivity increases then wages increase, since these are determined by marginal product alone; rents are unaffected

Equilibrium with no migration



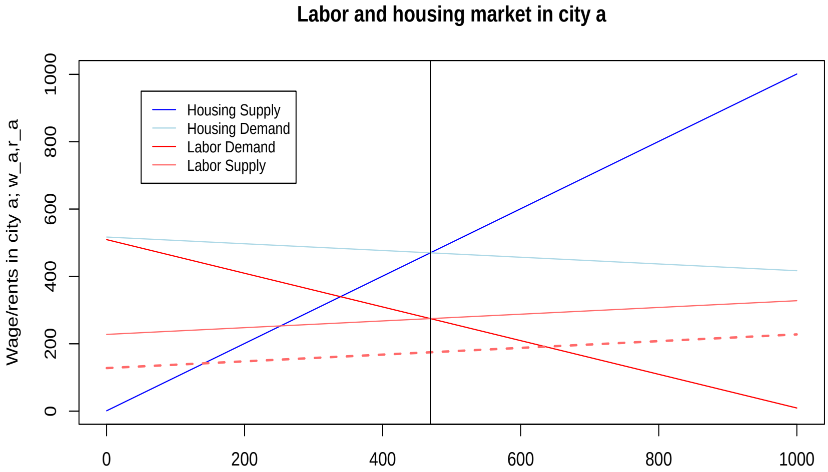
Equilibrium WITH migration

Labor supply and housing demand (population) are now elastic



Labor and housing markets are linked

Amenities increase, shifts labor supply, but new intersection not an equilibrium



Plotting demand and supply in rent and wage space

Since the markets are linked through population, we can substitute for population with supply or demand from other market

This allows plotting supply and demand versus rent r_c and wages w_c, somewhat similar to Roback

I insert the housing supply equation, $r_c = z + k_c N_c$, into labor demand and supply by substituting for population

$$r_b = -\frac{k_b}{1-h} w_b + z + \frac{k_b}{1-h}(X_b + ln(h)) \tag{PE1}$$

$$r_b = \frac{Nk_b}{Nk_b + s} w_b + \frac{Nk_b}{Nk_b + s}[-(w_a - r_a) + (A_b - A_a)] + \frac{s(k_b N_a + z)}{Nk_b + s} \tag{PE2}$$

Partial Equilibrium: Other City is Fixed

In $PE1$ and $PE2$ we are allowing the focal city's population to change, but fixing the population, wages, and rents of the other city

We can then solve for the partial equilibrium wages and rents by equation $PE1$ and $PE2$, which are simply lines. Rewrite as:

$$r_b = -\gamma_1 w_b + \Gamma_1 \tag{PE1: Labor Demand}$$

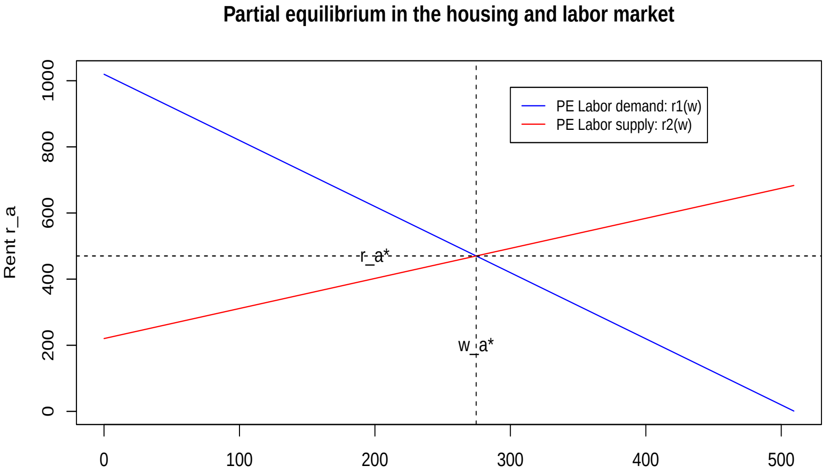
$$r_b = \gamma_2 w_b + \Gamma_2 \tag{PE2: Labor Supply}$$

Then partial equilibrium rent and wages are simply:

$$r_b^* = \frac{\Gamma_1 - \Gamma_2}{\gamma_1 + \gamma_2} \quad \text{and} \quad w_b^* = \frac{\gamma_2 \Gamma_1 + \gamma_1 \Gamma_2}{\gamma_1 + \gamma_2} \tag{PE3}$$

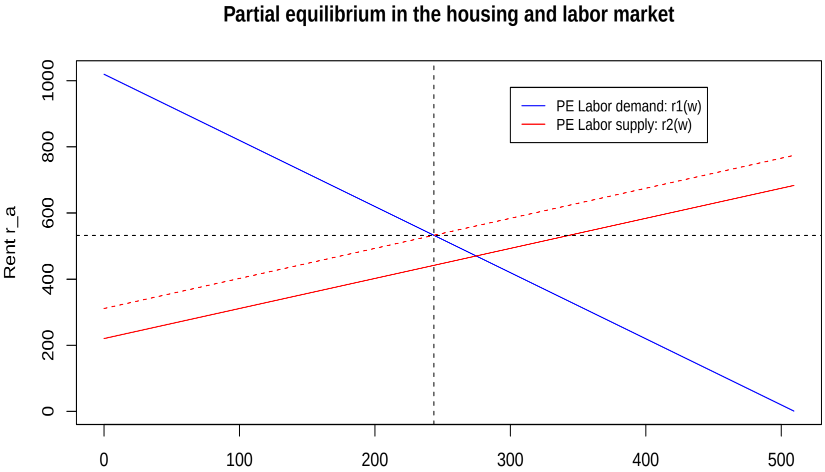
Partial equilibrium: population of *a* is mobile

Partial equilibrium because ignores city *a*'s effect on *b*



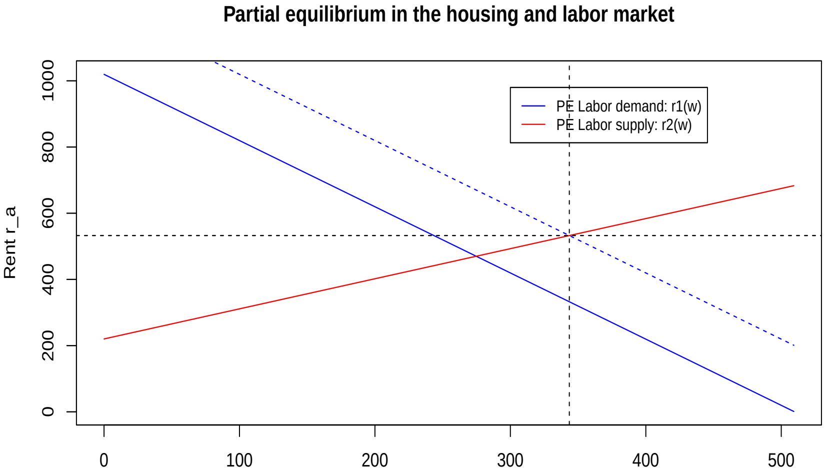
Increase in city amenities A_a

Population ↑, rent ↑, marginal product ↓, wages ↓



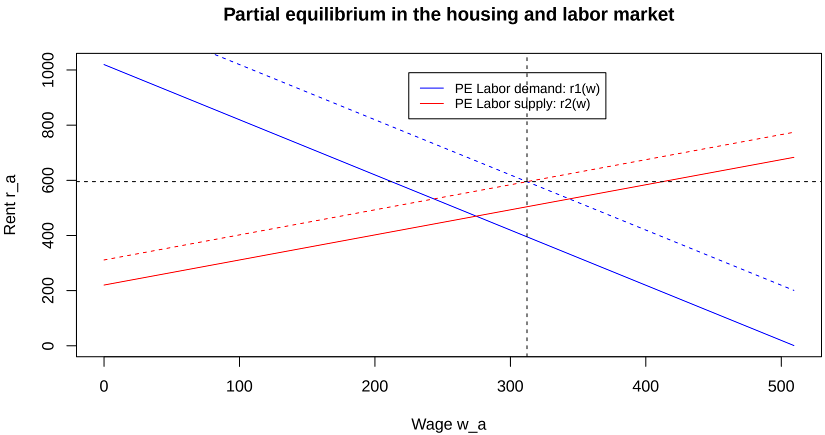
Increase in productivity X_a (productive amenity)

Wages ↑, Population ↑, rent ↑



Increase in amenities A_a and productivity X_a

Rent must increase since A_a and X_a raise r_a, but wage effect depends on parameters (just like Roback)



General equilibrium

Our analysis has allowed for N_a to change, but kept N_b fixed, as well as w_b, r_b

However, if $N_a \uparrow$ then $N_b \downarrow$, which affects wages and rents in city *b*

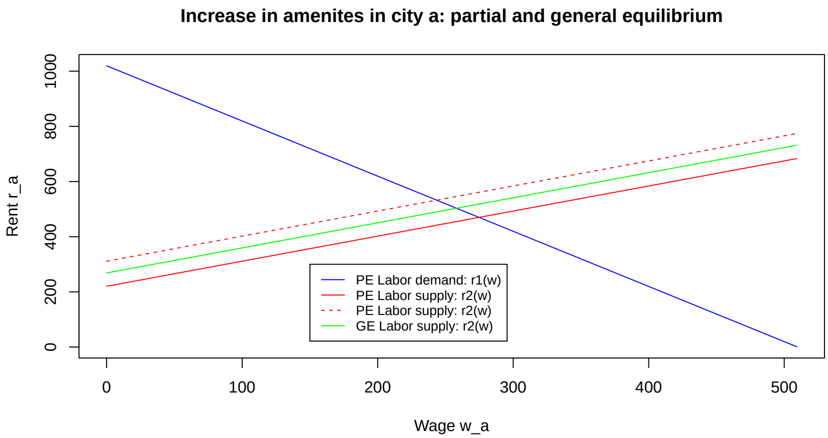
Therefore, in general equilibrium a change in *a* that cause a flow from *b* will also affect *b*

This can offset some of the partial equilibrium effect, because the utility difference between cities will not be as large, thus limiting population flow

For example, if amenities increase in *a* then people from *b* migrate to *a*, which decreases rents in *b* and raises wages in *b* through high marginal productivity of labor

Increase in amenities A_a: general equilibrium vs partial equilibrium

GE effect is smaller because city *b* utility increases, decreasing outflow



Concluding thoughts on Moretti model

Simple two city framework that helps to better understand Roback model

Can also extend to incorporate other effects (land in production, agglomeration, tradable vs nontradable goods, skill levels, etc...)

By changing distribution of ϵ_{ic} term can allow for many cities (however, model is less analytically tractable)

Generally, a nice workable framework for thinking about empirical applications

References

1. David Card's lecture notes on local labor markets: http://eml.berkeley.edu/ webfac/card/e250a_f16/lecture8-2016.pdf
2. Albouy, David, "Are Big Cities Bad Places to Live? Estimating Quality of Life across Metropolitan Areas.", Working Paper (JPE revision, davidalbouy.net)
3. Zheng, Siqi, Fu, Yuming, Liu Hongyu, "Demand for Urban Quality of Living in China: Evolution in Compensating Land-Rent and Wage-Rate Differentials." *Journal of Real Estate Finance and Economics*, 2009.
4. Huang, Xuan and Lanz, Bruno, "The Value of Air Quality in Chinese Cities: Evidence from Labor and Property Market Outcomes," *Environmental and Resource Economics*, 2017.
5. Glenn C. Blomquist, Mark C. Berger and John P. Hoehn, "New Estimates of Quality of Life in Urban Areas." *The American Economic Review*,1988