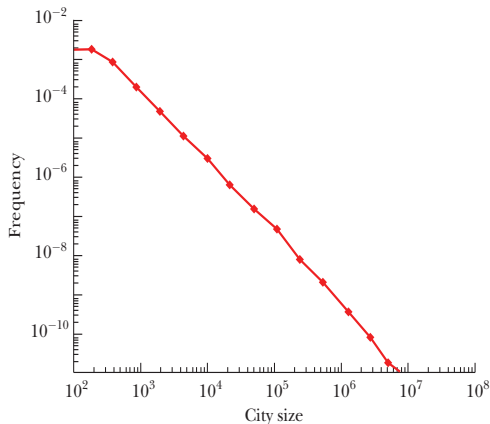


Zipf's Law in UK: Gabaix 2016

Density Function of City Sizes (Agglomerations) for the United Kingdom



Source: Rozenfeld et al. (2011).

Notes: We see a pretty good power law fit starting at about 500 inhabitants. The Pareto exponent is actually statistically non-different from 1 for size $S > 12,000$ inhabitants.

Why is this important?

This empirical relationship is so strong $R^2 \sim 1$ some economists (Gabaix) propose that any system of cities model which tries to explain the data must lead to this regularity

For example, one of the classic models for cities (Henderson, 1974) does not lead to Zipf's distributions

Gabaix JEP 2016 considers this one of the few “non-trivial and true” results of economics

Note: this paper also discusses other power laws in economics and shows that firm size distribution is Zipf ($\zeta = -1$)

What explains Zipf's Law?

Say we start out with a set of cities of all different population sizes (some big, some small, etc...)

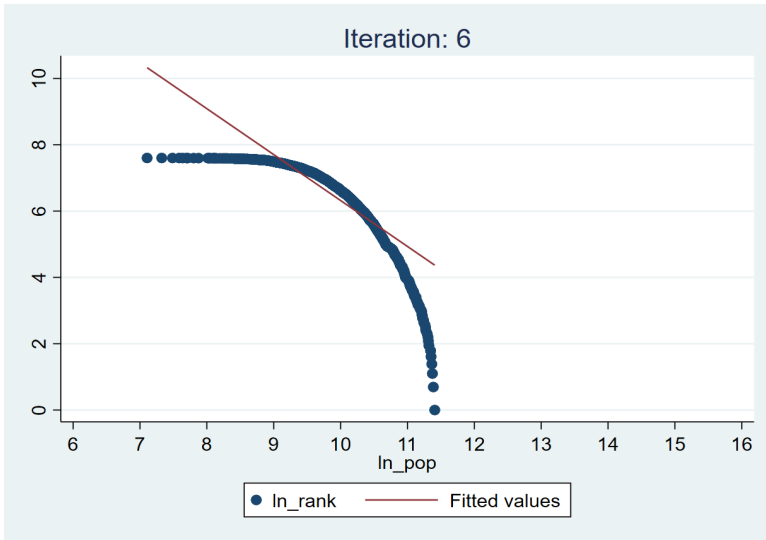
If these cities grow and shrink randomly—the population growth rate does not depend on the initial population size population level—then the distribution will converge to a power law

Technical note: there must also be a lower bound—cities cannot shrink below some fixed population

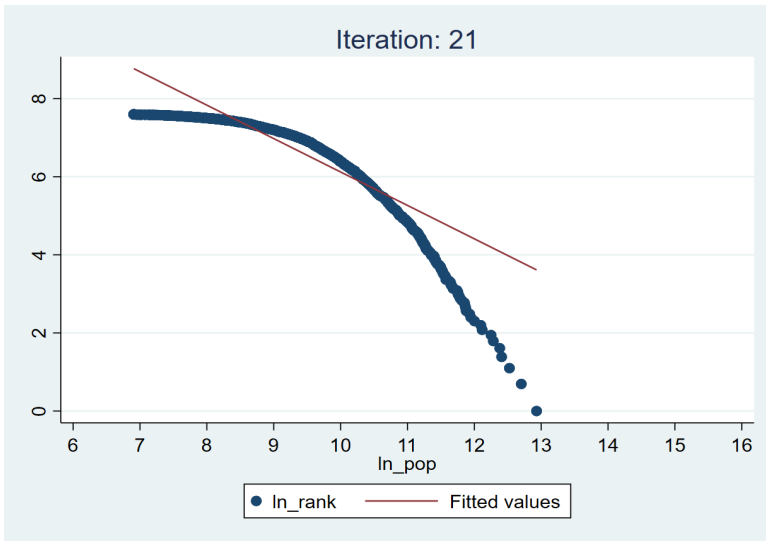
This exponent of this power law depends on the growth process, *but*, Gabaix (1999) showed that if the total population is fixed the exponent will converge to 1: Zipf's Law

Here is a simulation demonstration

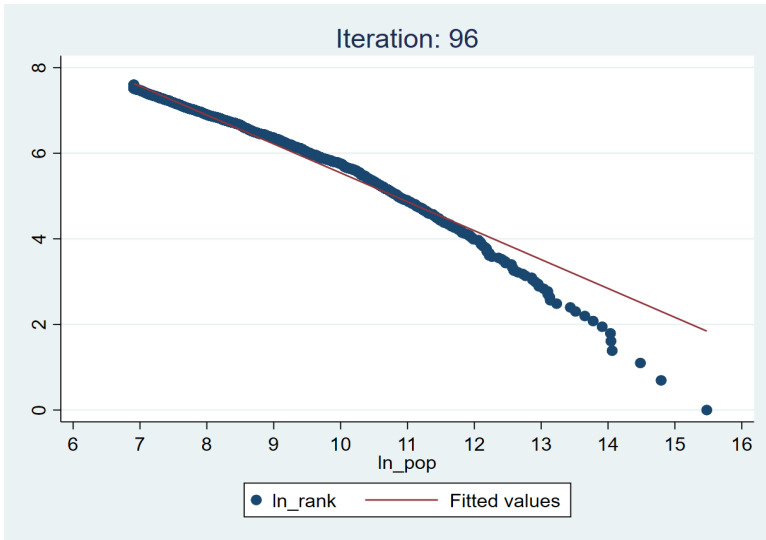
Random Growth Demonstration



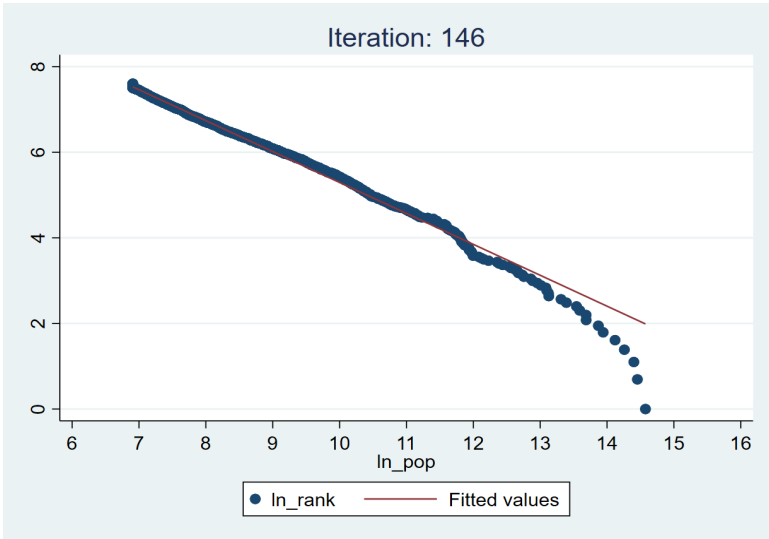
Random Growth Demonstration



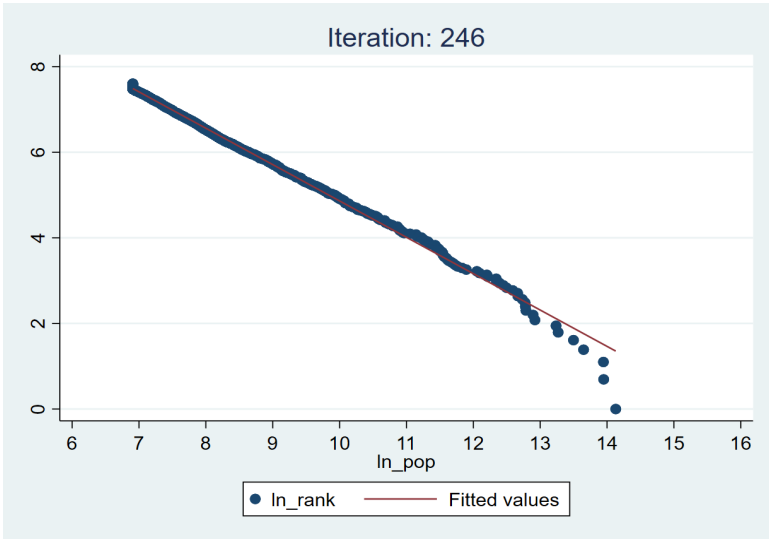
Random Growth Demonstration



Random Growth Demonstration



Random Growth Demonstration



Why Would Cities Grow Randomly?

Random growth is consistent with constant returns to scale: doubling inputs (ex: population) leads to double outputs, growth rate is same across cities of different sizes

But, lots of theories suggest city growth is affected by characteristics of the city (human capital levels, geography, amenities)

Further, empirical evidence suggests US cities with higher human capital have grown faster (Glaeser et. al. 1995, Shapiro 2006); we will see that effect seems to be very strong in China (Chauvin et. al. 2017)

This evidence seems to contradict random growth, although it's possible human capital effects eventually mean revert

There are also other models that can generate a Zipf distribution; see Behrens, Duranton, Robert-Nicoud (2013) for one example

Ongoing Line of Research

Zipf's Law continues to be extensively studied

Some discussion over exact form (power law vs log normal distribution, see Eeckhout 2004)

Much work on cross-country comparisons, including this paper

Additional work on how to define a city (Rozenfeld, Rybski, Gabaix, Makse, AER 2011)

How universal is Zipf's Law—does it hold among small geographies? (Holmes and Lee, 2010)

Lee and Li (JUE 2013) show that Zipf's Law can result from product of multiple random factors

Implies that cannot use Zipf's Law to test system of cities models since even if a single model does not yield Zipf's Law it may when combined with other models (and we do not usually assume our models are exhaustive)

Back to CGMT: Zipf's Law

CGMT look for evidence of Zipf's Law and Gibrat's Law in country sample

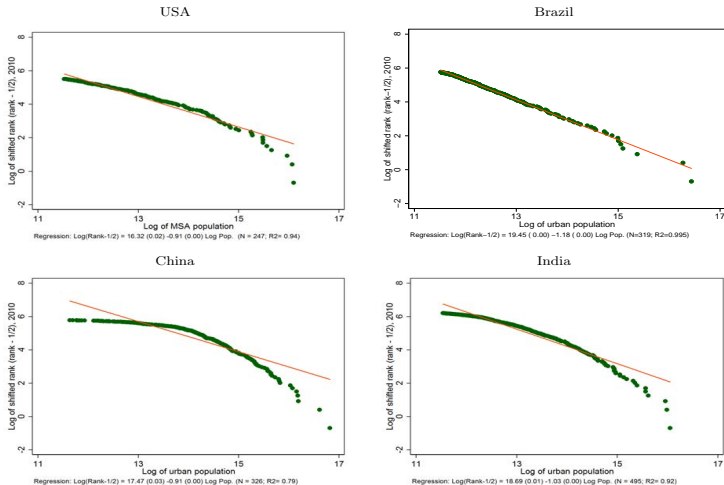
Focus is on simplest methodologies and use of data comparable across countries

Test Zipf's Law with standard regression of $\log(\text{Rank})$ on $\log(\text{Pop})$ —for econometric reasons they use $\log(\text{Rank}-0.5)$

Test Gibrat's Law by regressing population growth on initial population

Zipf's Law, CGMT

Figure 2: Zipf's Law. Urban populations and urban population ranks, 2010



Note: Regression specifications and standard errors based on Gabaix and Ibragimov (2011). Samples restricted to areas with urban population of 100,000 or larger.

Sources: See data appendix.

Zipf Law Results

US has coefficient close to -1, consistent with past findings

In Brazil, fit is linear but slope is -1.18—steeper than Zipf's Law

China has very non-linear shape—does not fit straight line power law pattern

China has too *few* large cities to be consistent with Zipf's Law

India is also somewhat curved but closer to US fit

Authors also do KS test on distributions, find China's distribution particularly distinct from other three countries

Gibrat's Law Regressions

Table 4: Gibrat's Law: Urban population growth and initial urban population

	USA (MSAs)	Brazil (Microregions)	China (Cities)	India (Districts)
1980 - 2010	0.009 (0.020) N=217 R2=0.001	-0.038 (0.023) N = 144 R2 = 0.015	-0.447*** (0.053) N=187 R2=0.280	-0.052** (0.023) N=237 R2=0.021
1980 - 1990	0.008 (0.008) N=217 R2=0.004	-0.026** (0.013) N = 144 R2 = 0.020	-0.310*** (0.054) N=187 R2=0.151	0.063* (0.034) N=237 R2=0.015
1990 - 2000	0.014** (0.007) N=217 R2=0.019	0.001 (0.010) N = 144 R2 = 0.000	-0.308*** (0.036) N=187 R2=0.280	0.005 (0.020) N=237 R2=0.00
2000 - 2010	0.012** (0.006) N=217 R2=0.018	0.006 (0.006) N = 144 R2 = 0.006	0.019 (0.021) N=187 R2=0.005	-0.013 (0.015) N=237 R2=0.004

Note: All figures reported correspond to area-level regressions of the log change in urban population on the log of initial urban populations in the specified period. Regression restricted to areas with urban population of 100,000 or more in 1980. Robust standard errors in parentheses.

*** p<0.01, ** p<0.05, * p<0.1

Sources: See data appendix.

