

Chauvin, Glaeser, Ma, Tobio, JUE 2017

Chauvin, Glaeser, Ma, Tobio (CGMT) note that most empirical work in urban economics has focused on the US

Urban empirical work in other countries beside US focused on developed countries (mostly Europe)

General question of CGMT: do all the spatial patterns documented in developed countries hold for developing nations?

Examine US, Brazil, India, and China

Specifically look at 1) Zipf's Law 2) Spatial Equilibrium evidence 3) Agglomeration Externalities evidence

They do not look at *within* city patterns, focus of next couple classes

A Remarkable Fit! What is going on?

While the largest cities were off the line, this is generally a remarkable fit!

In economics, we *never* have R-squared values of 0.98 (if you find one, you have made a mistake).

This fit implies that if we know only the rank of the city, we can make a very accurate prediction for the population (outside of the top 7 cities)

Further, we found $\ln(Rank) = \alpha + -1.035 * \ln(Population)$

Exponentiate both sides: $Rank = e^{\alpha} * Pop^{-1.035}$, or $Pop \approx e^{\alpha} / R$

This implies that the population of every city is proportional to its rank. The population of the largest city is $e^{\alpha} / 1$, the second largest city is $e^{\alpha} / 2$, third is $e^{\alpha} / 3$.

Alternatively, the population of the second largest city is half the population of the largest, the pop of the third is a third the population of the largest, the population of the N th city is $1/N$ times the population of the largest... **What is going on?**

Urbanization in China: Discussion of Chauvin, Glaeser, Ma, Tobio (2017)

└ Zipf

└ Power Laws

Power Laws

Let $p(x)$ be the probability of observing a variable with a value equal to x , such as

a height of 163cm ($x = 163$), or a city size of one million people ($x = 1000000$)

If this probability takes the form $p(x) = C \cdot x^{-(\zeta+1)}$ then the distribution of this variable follows a power law.

The C term is just a constant and not important; the key term is ζ , with $\zeta \geq 0$. Since this exponent is negative, larger values of x are less likely to be observed.

$$Pr(X \geq x) = \frac{C}{\zeta} x^{-\zeta} = a \cdot x^{-\zeta} \quad (1)$$

If observation x_r is the r largest observation (rank), then $Pr(X \geq x_r) \sim r$

Thus $r \sim ax^{-\zeta}$, or our plot: $\ln(\text{Rank}) = \ln(a) - \zeta \cdot \ln(\text{Population})$

1. C is unimportant in the sense that it is determined from the requirement that the probability must sum to 1: $\int_{x_{min}}^{\infty} p(x) dx = 1$. Given that the range goes to ∞ , we must assume $\zeta > 0$, which yields $C = (\zeta)x_{min}^{\zeta}$, see Appendix A in Newman.
2. $Pr(X \geq x) = C \int_{s=x}^{\infty} s^{-(\zeta+1)} ds = \frac{C}{\zeta} x^{-\zeta}$
3. Consider drawing from a set of 100 observations. The probability of drawing a value \geq to the largest value is 1/100. The probability of getting a value \geq to the second largest value is 2/100, and so on.

Variables that Follow Power Laws are *Scale Free*

The probability of observing a variable with a value equal to x is:

$$p(x) = C * x^{-(\zeta+1)}$$

How much more likely are we to observe x compared to $2x$?

$$\frac{p(x)}{p(2x)} = \frac{C * x^{-(\zeta+1)}}{C * (2x)^{-(\zeta+1)}} = (1/2)^{-(\zeta+1)}$$

How much more likely are we to observe $1000x$ compared to $2000x$?

$$\frac{p(1000x)}{p(2000x)} = \frac{C * (1000x)^{-(\zeta+1)}}{C * (2000x)^{-(\zeta+1)}} = (1/2)^{-(\zeta+1)}$$

This is a very unique and unusual property. Say cities with 1000 people are four times more common than cities with 2000 people. Then it is also true that cities of one million people are four times more common than cities of two million people.

When a variable follows a power law, we see the same pattern at very small scales as we do at very large scales

Why is this important?

This empirical relationship is so strong $R^2 \sim 1$ some economists (Gabaix) propose that any system of cities model which tries to explain the data must lead to this regularity

For example, one of the classic models for cities (Henderson, 1974) does not lead to Zipf's distributions

Gabaix JEP 2016 considers this one of the few “non-trivial and true” results of economics

Note: this paper also discusses other power laws in economics and shows that firm size distribution is Zipf ($\zeta = 1$)

What explains Zipf's Law?

Say we start out with a set of cities of all different population sizes (some big, some small, etc...)

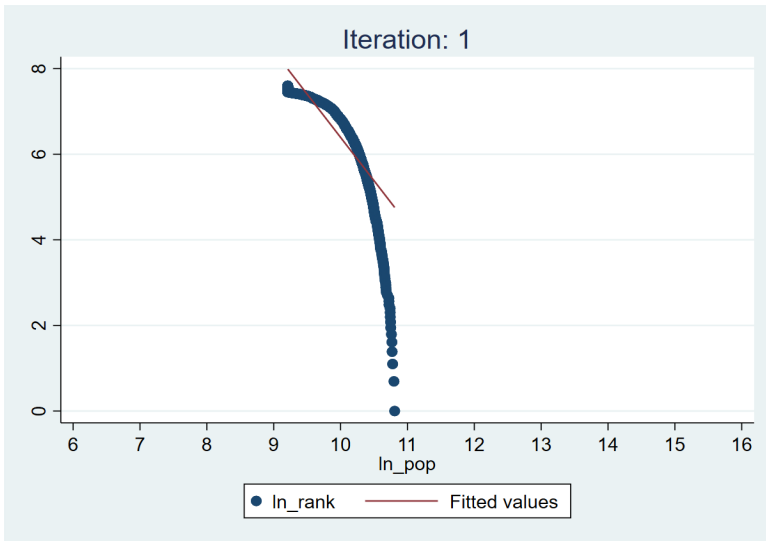
If these cities grow and shrink randomly—the population growth rate does not depend on the initial population size population level—then the distribution will converge to a power law

Technical note: there must also be a lower bound—cities cannot shrink below some fixed population

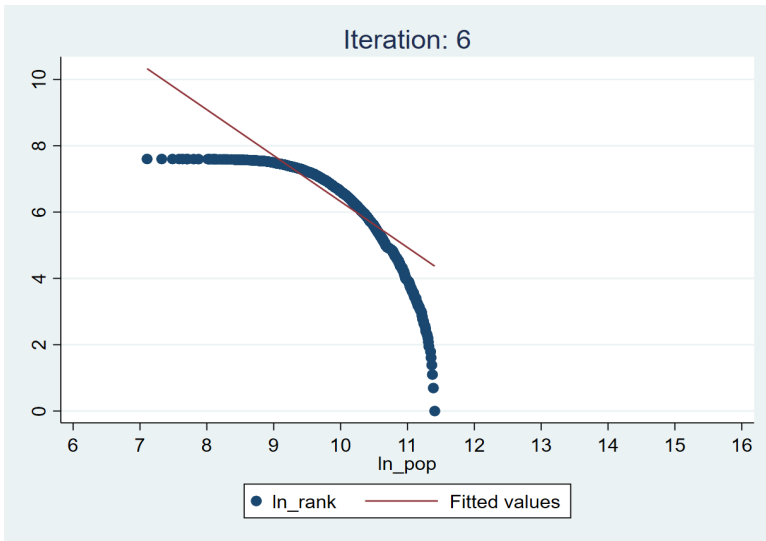
This exponent of this power law depends on the growth process, *but*, Gabaix (1999) showed that if the total population is fixed the exponent will converge to 1: Zipf's Law

Here is a simulation demonstration

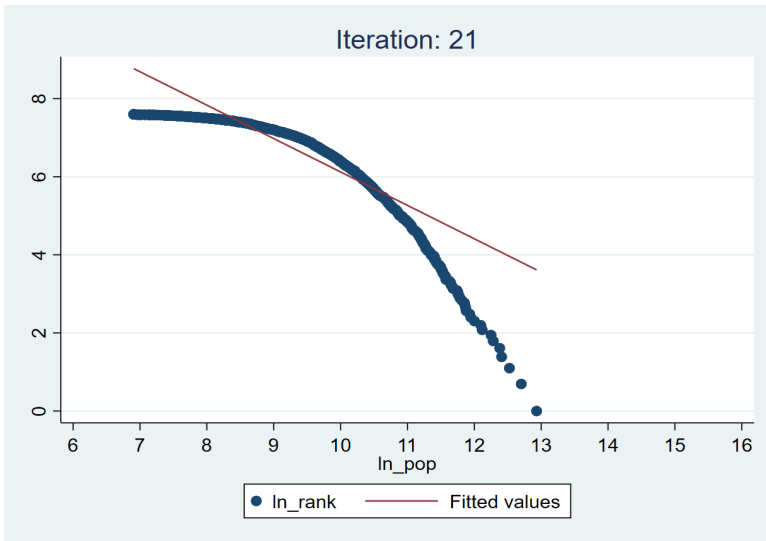
Random Growth Demonstration



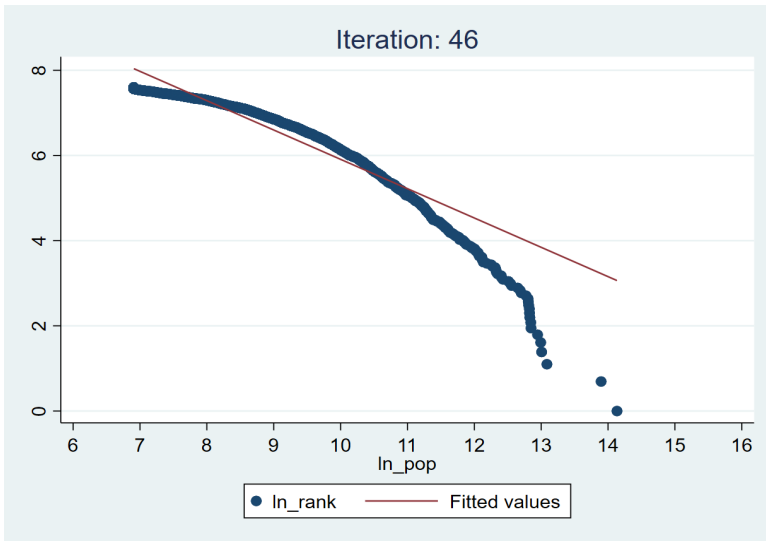
Random Growth Demonstration



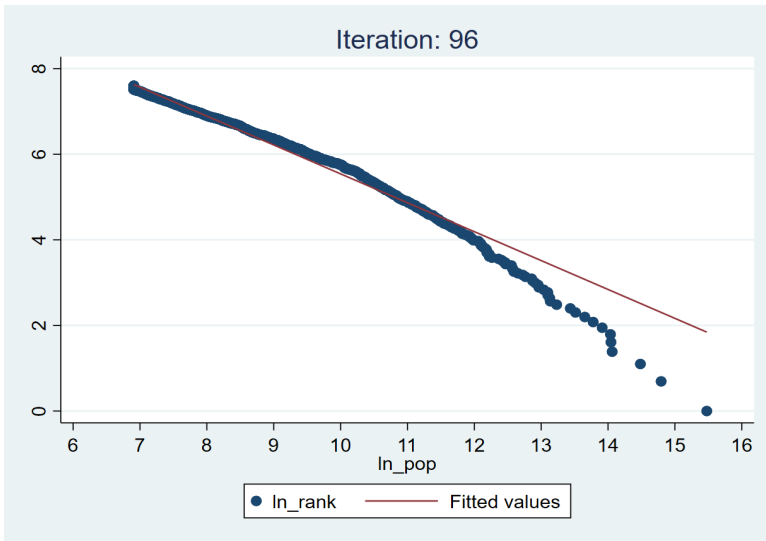
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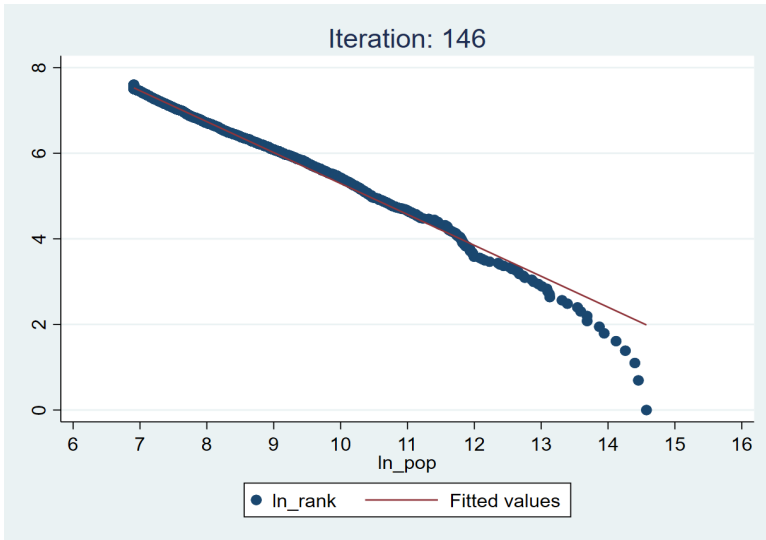
Random Growth Demonstration



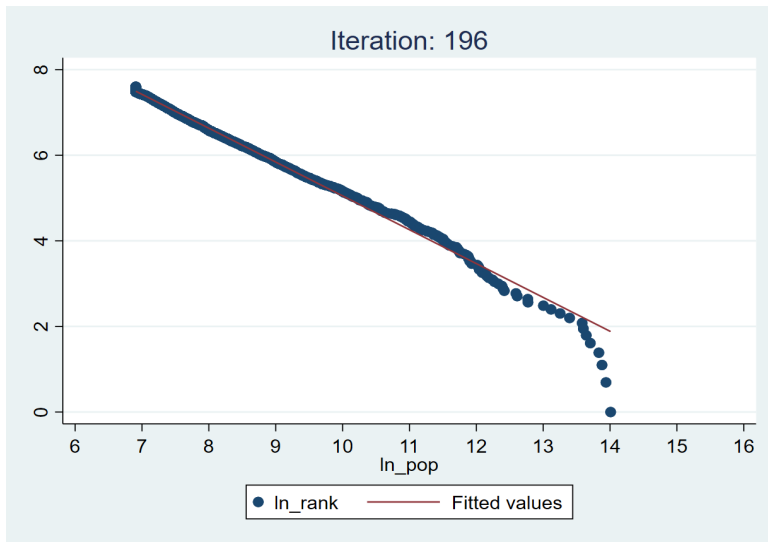
Random Growth Demonstration



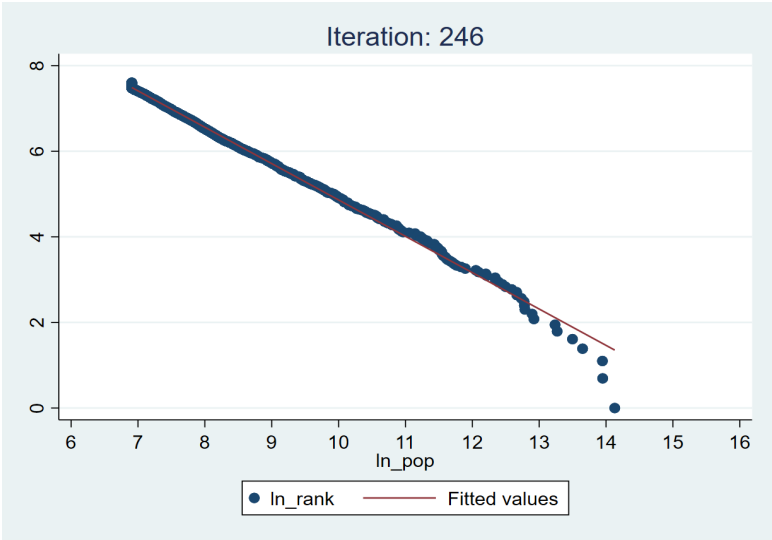
Random Growth Demonstration



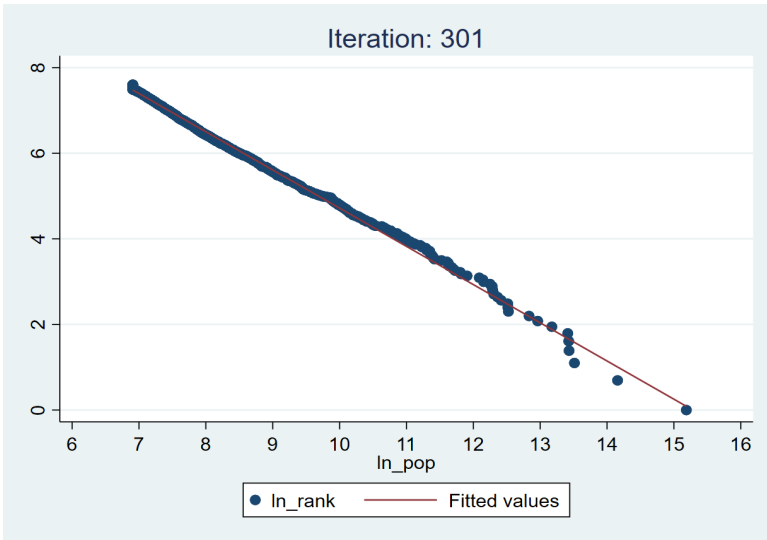
Random Growth Demonstration



Random Growth Demonstration



Random Growth Demonstration



Why Would Cities Grow Randomly?

Random growth is consistent with constant returns to scale: doubling inputs (ex: population) leads to double outputs, growth rate is same across cities of different sizes

But, lots of theories suggest city growth is affected by characteristics of the city (human capital levels, geography, amenities)

Further, empirical evidence suggests US cities with higher human capital have grown faster (Glaeser et. al. 1995, Shapiro 2006); we will see that effect seems to be very strong in China (Chauvin et. al. 2017)

This evidence seems to contradict random growth, although it's possible human capital effects eventually mean revert

There are also other models that can generate a Zipf distribution; see Behrens, Duranton, Robert-Nicoud (2013) for one example

Ongoing Line of Research

Zipf's Law continues to be extensively studied

Some discussion over exact form (power law vs log normal distribution, see Eeckhout 2004)

Much work on cross-country comparisons, including this paper

Additional work on how to define a city (Rozenfeld, Rybski, Gabaix, Makse, AER 2011)

How universal is Zipf's Law—does it hold among small geographies? (Holmes and Lee, 2010)

Lee and Li (JUE 2013) show that Zipf's Law can result from product of multiple random factors

Implies that cannot use Zipf's Law to test system of cities models since even if a single model does not yield Zipf's Law it may when combined with other models (and we do not usually assume our models are exhaustive)

Back to CGMT: Zipf's Law

CGMT look for evidence of Zipf's Law and Gibrat's Law in country sample

Focus is on simplest methodologies and use of data comparable across countries

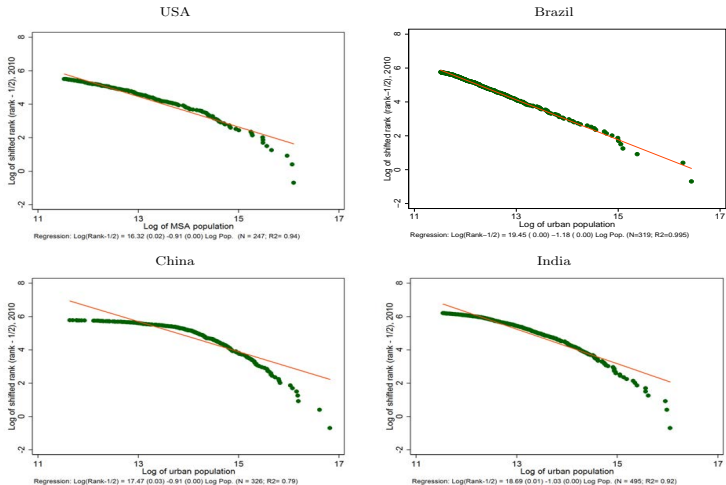
Test Zipf's Law with standard regression of $\log(\text{Rank})$ on $\log(\text{Pop})$ —for econometric reasons they use $\log(\text{Rank}-0.5)$

Test Gibrat's Law by regressing population growth on initial population

1. If the equation is $y = ax^{-\zeta}$ then taking logs and regressing leads to a biased estimator of ζ . In Gabaix and Ibragimov 2011 they show that this bias is greatly reduced by simply subtracting 1/2 within the log function, $\log(y - 0.5)$. This is still an approximation and not as accurate as estimating ζ with non-linear regression, but much simpler.

Zipf's Law, CGMT

Figure 2: Zipf's Law. Urban populations and urban population ranks, 2010



Note: Regression specifications and standard errors based on Gabaix and Ibragimov (2011). Samples restricted to areas with urban population of 100,000 or larger.

Sources: See data appendix.

Zipf Law Results

US has coefficient close to -1, consistent with past findings

In Brazil, fit is linear but slope is -1.18—steeper than Zipf's Law

China has very non-linear shape—does not fit straight line power law pattern

China has too *few* large cities to be consistent with Zipf's Law

India is also somewhat curved but closer to US fit

Authors also do KS test on distributions, find China's distribution particularly distinct from other three countries

Gibrat's Law Regressions

Table 4: Gibrat's Law: Urban population growth and initial urban population

	USA (MSAs)	Brazil (Microregions)	China (Cities)	India (Districts)
1980 - 2010	0.009 (0.020) N=217 R2=0.001	-0.038 (0.023) N = 144 R2 = 0.015	-0.447*** (0.053) N=187 R2=0.280	-0.052** (0.023) N=237 R2=0.021
1980 - 1990	0.008 (0.008) N=217 R2=0.004	-0.026** (0.013) N = 144 R2 = 0.020	-0.310*** (0.054) N=187 R2=0.151	0.063* (0.034) N=237 R2=0.015
1990 - 2000	0.014** (0.007) N=217 R2=0.019	0.001 (0.010) N = 144 R2 = 0.000	-0.308*** (0.036) N=187 R2=0.280	0.005 (0.020) N=237 R2=0.00
2000 - 2010	0.012** (0.006) N=217 R2=0.018	0.006 (0.006) N = 144 R2 = 0.006	0.019 (0.021) N=187 R2=0.005	-0.013 (0.015) N=237 R2=0.004

Note: All figures reported correspond to area-level regressions of the log change in urban population on the log of initial urban populations in the specified period. Regression restricted to areas with urban population of 100,000 or more in 1980. Robust standard errors in parentheses.

*** p<0.01, ** p<0.05, * p<0.1

Sources: See data appendix.

Discussion of Zipf and Gibrat Results

US and Brazil fit well but India doesn't and China is large outlier

China data also not consistent with Gibrat's Law; shows mean reversion, smaller cities grow faster

Authors suggest China may still be far from steady state spatial equilibrium

Further suggest that government role in migration could alter market-based city distribution

- Note: China has active population management policies, including population caps as part of “master urban plans” (ex: Shanghai 25m in 2035), seems reasonable that these policies could lead to deviations from Gibrat's Law

Authors suggest that possible in long-run “China's urban populations will be much more skewed towards ultra large areas like Beijing and Shanghai.”

Dingel, Miscio, and Davis, JUE 2020

In US and Europe, metropolitan areas (economically connected parts of cities) are defined with commuting flows

In China and India, these spatial definitions are not available and so researchers usually use administrative (politically defined) areas

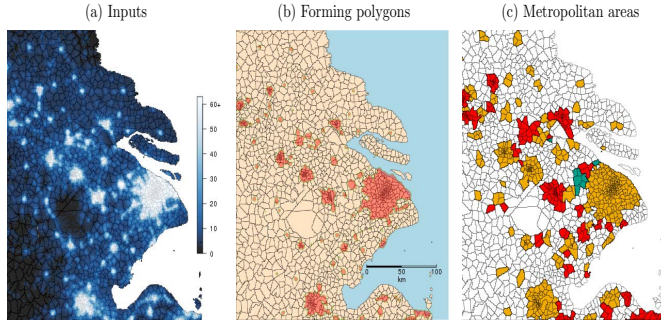
Problem: administrative areas may not correspond to economic areas, leading to strange results in analysis. For ex, DDM point out that Foshan and Guangzhou are only 18 miles apart and connected by a subway, yet are still defined as separate prefectures.

In “Cities, Lights, and Skills in Developing Economies,” authors redo rank/size regressions (and additional analysis) using spatial units defined by satellite data on night lights intensity

With their definition of metro areas, Chinese cities conform to a power law (but with a coefficient greater than one)

Using night lights to defined metropolitan areas in China

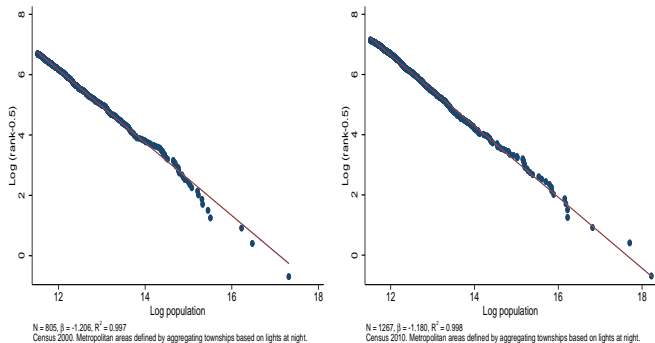
Figure 1: Building metropolitan areas by aggregating smaller units based on lights at night



NOTES: This figure illustrates our procedure for combining satellite imagery of lights at night with administrative spatial units to build metropolitan areas. These panels depict a portion of the eastern coast of China in 2000. The administrative spatial units are townships. The polygons in the middle panel are areas of contiguous light brighter than 30. Aggregating the townships that intersect these polygons produces the metropolitan areas depicted in the right panel. Adjacent townships are often assigned to distinct metropolitan areas.

Zipf's Law for China using Metros defined with night lights

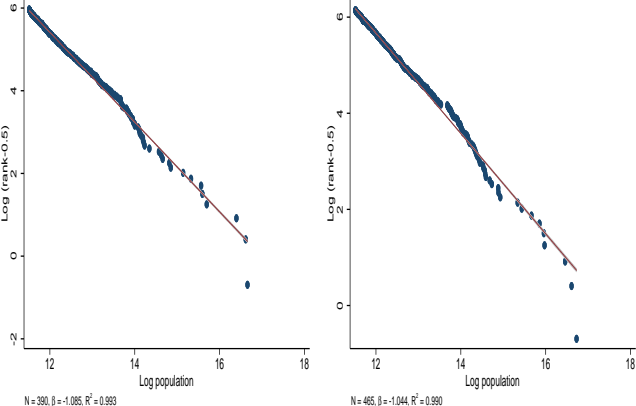
Figure 7: China's city-size distribution with night-lights-based units, 2000 and 2010



NOTES: The sample is Chinese metropolitan areas with population greater than 100,000. Metropolitan areas defined by aggregating townships in areas of contiguous night lights with intensity greater than 30. Left panel depicts 2000; right panel 2010.

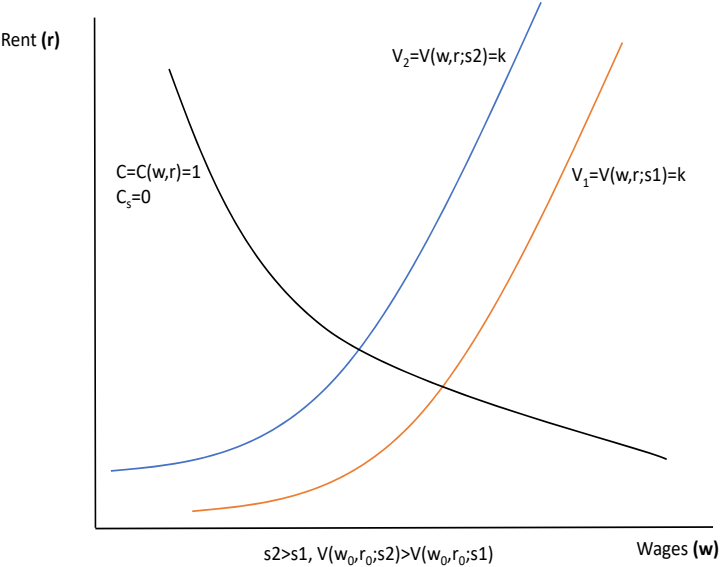
Zipf's Law for India using Metros defined with night lights

Figure 8: India's city-size distribution, urban agglomerations, 2001 and 2011



Spatial Equilibrium

Rosen-Roback Model: Consumer Amenity Only



Prices and Wages: Cobb-Douglas

Say people have utility $U = A * H^\alpha C^{1-\alpha}$ and after-tax wages $(1 - t) * W$

Then indirect utility function, with constant K , is $V = K * A * (1 - t)W * P_H^{-\alpha}$

Take logs and re-arrange: $\ln(P_H) = \frac{1}{\alpha} (\ln(K/V) + \ln((1 - t) * W) + \ln(A))$, or:

$$\text{Log}(H\text{Price}_i) = \frac{1}{\alpha} (\text{Constant} + \text{Log}(Wage_i) + \text{Log}(Amenities_i)) \tag{1}$$

Then $\partial E[\text{Log}(H\text{Price}_i)|X]/\partial \text{Log}(Wage_i) = \frac{1}{\alpha} \left(1 + \frac{\text{Cov}(\text{Log}(wage), \text{Log}(Amenities))}{\text{Var}(\text{Log}(Wage))} \right)$

If $\text{Cov}(\text{Log}(wage), \text{Log}(Amenities)) = 0$ then coeff=1/α; US households spend α = 1/3 of income on housing so coeff=3 (China's α = 1/10)

Urbanization in China: Discussion of Chauvin, Glaeser, Ma, Tobio (2017)

Spatial Equilibrium

Wages and Rents Regressions

Wages and Rents Regressions

Table 1
Regressions of local prices on wages, 2005

	USA (MSAs)	Spain (Microregions)	China (Cities)	India (Districts)	USA (MSAs)	China (Cities)
	log of rents			log of prices		
Average log wage	1.22*** (0.08) N = 29 M R ² = 0.28	1.93*** (0.04) N = 839 K R ² = 0.50	0.853*** (0.02) N = 6.5 K R ² = 0.50	-0.664 (0.25) N = 148 R ² = 0.34	1.52*** (0.02) N = 36 M R ² = 0.39	1.12*** (0.02) N = 243 K R ² = 0.12
Average log wage residual	1.02*** (0.08) N = 29 M R ² = 0.20	1.907*** (0.04) N = 839 K R ² = 0.50	1.03*** (0.02) N = 6.5 K R ² = 0.50	-0.89 (0.40) N = 148 R ² = 0.34	2.807*** (0.29) N = 36 M R ² = 0.40	1.09*** (0.12) N = 243 K R ² = 0.15
Controlling characteristics	Yes	Yes	Yes	Yes	Yes	Yes

Note: Regressions at the urban household level, restricted to areas with urban population of 100,000 or more. All regressions include a constant. Standard errors clustered at the area level in parentheses. *** p < 0.01, ** p < 0.05, * p < 0.1.
Source: See data appendix.

1. p9: “The first row shows results when we define income as the average of the logarithm of income in the area. The second row instead uses the average of the residual from a regression of the logarithm of wages on human capital characteristics.”
2. NS: Note that authors clustered standard errors at area level, thus 29 million is not relevant observation count.

Migration and Mobility

Table 7: Percentage of the population living in a different locality five years ago

	USA			Brazil		
	1990	2000	2010	1991	2000	2010
Migrants in the last 5 years (% of population)	21.3%	21.0%	13.8%	9.5%	9.1%	7.1%
From same state/prov., different county / dist.	9.7%	9.7%	6.7%	6.0%	5.4%	4.5%
From different state/province	9.4%	8.4%	5.6%	3.5%	3.6%	2.4%
From abroad	2.2%	2.9%	1.5%	0.04%	0.1%	0.14%
	China		India			
	2000	2010	1993	2001	2011	
Migrants in the last 5 years (% of population)	6.3%	12.8%	1.9%	2.6%	2.0%	
From same state/prov., different county / dist.	2.9%	6.4%	1.3%	1.5%	1.2%	
From different state/province	3.4%	6.4%	0.6%	1.0%	0.8%	
From abroad	N/A	N/A	0.02%	0.1%	0.03%	

Sources: See data appendix

Agglomeration and Human Capital in Cities

Productivity in Big Cities: Agglomeration Externalities

One of the most fundamental ideas in urban economics is that concentrating workers leads to higher productivity

Without such a force, the only way to explain the existence of cities is through heterogeneity in land productivity (very hard story to justify Beijing/Shanghai)

Extensive and deep empirical work in urban economics documents agglomeration externalities, simplest form regresses log wage on log population (Melo et. al. 2009 meta analysis suggests elasticity of 0.02-0.1)

Lots of recent work on agglomeration benefits of concentrating high skilled workers (ex: Moretti papers)

CGMT focus on 1) population (density) on wages 2) area education on wages and pop. and wage growth

Agglomeration Externalities: Real Income

Table 9: Real income and agglomeration, 2010

	USA (MSAs)	Brazil (Microregions)	China (Cities)	India (Districts)
	Log real wage	Log real wage	Log real wage	Log real wage
OLS regressions				
Log of urban population	0.0190** (0.00916)	0.011 (0.010)	-0.0313 (0.0307)	0.0688** (0.0298)
	R2= 0.067	R2=0.310	R=0.174	R2=0.240
Log of density	0.0219 (0.0134)	0.002 (0.007)	0.0516** (0.0166)	0.0691*** (0.0213)
	R2=0.068	R2=0.309	R2=0.179	R2=0.244
Observations	28.5M	2,172 K	147K	2,102
IV1 regressions				
Log of urban population	0.0209** (0.0102)	0.009 (0.010)	-0.0664 (0.0485)	0.116 (0.0927)
	R2=0.068	R2 = 0.310	R2=0.174	R2=0.243
Log of density	0.0230* (0.0134)	0.001 (0.007)	0.0345* (0.0175)	0.0647** (0.0255)
	R2=0.068	R2 = 0.309	R2=0.179	R2=0.241
Observations	28.5M	2,172 K	143K	1,649
IV2 regressions				
Log of urban population	0.0466** (0.0190)	-0.017 (0.016)	0.0648 (0.0743)	0.208** (0.0840)
	R2=0.065	R2 = 0.305	R2=0.161	R2=0.244
Log of density	0.0419** (0.0163)	-0.008 (0.008)	0.0665 (0.0625)	0.0512* (0.0263)
	R2=0.067	R2 = 0.307	R2=0.179	R2=0.241
Observations	28.5M	1,998 K	112K	1,141
Educational attainment controls	Yes	Yes	Yes	Yes
Demographic controls	Yes	Yes	Yes	Yes

Note: Regressions at the individual level, restricted to urban prime-age males in areas with urban population of 100,000 or more. All regressions include a constant.

Robust standard errors in parentheses.

*** p<0.01, ** p<0.05, * p<0.1

Sources: See data appendix.

CGMT Concluding Thoughts

1. US and Brazil follow Zipf; China and India have too few large cities
2. Relationship between income and rents similar in US, Brazil, and China; not India
3. Generally, spatial equilibrium not as strong a fit in China as US and Brazil; authors suggest this might reflect hukou system
4. Connection between human capital and area success (growth) higher in Brazil, China, India compared to US
5. Overall, suggest spatial equilibrium model appropriate for Brazil, China, US, but not India

