Cities and Product Variety

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Variety and cities

Why study product variety in cities?

- Consumer cities literature suggests consumption amenities attract people to cities (Glaeser et al, 2001)
    - Unique consumption goods of cities are non-tradeable
    - The types and range of these goods is a key consumption amenity of cities
- Product differentiation provides insight into how firms compete
    - If cities show markedly higher differentiation it may suggest a different competitive environment from smaller places

Very little evidence of non-tradeable variety across cities

Question: do cities have greater non-tradeable variety and if so, why?

Main Question

How does demand density—aggregation of demand in geographic space–affect product variety?

Specifically, for non-tradable consumer goods–bars, music venues, hair salons, health clubs, specialty boutiques, restaurants–how does a city's population and land area affect the variety available?

Two forces:

- Scale: greater populations support greater variety
- Transportation cost: dispersed consumers lower demand for any firm

This paper: show how these competing forces affect restaurant variety in US cities

Describing consumption good variety

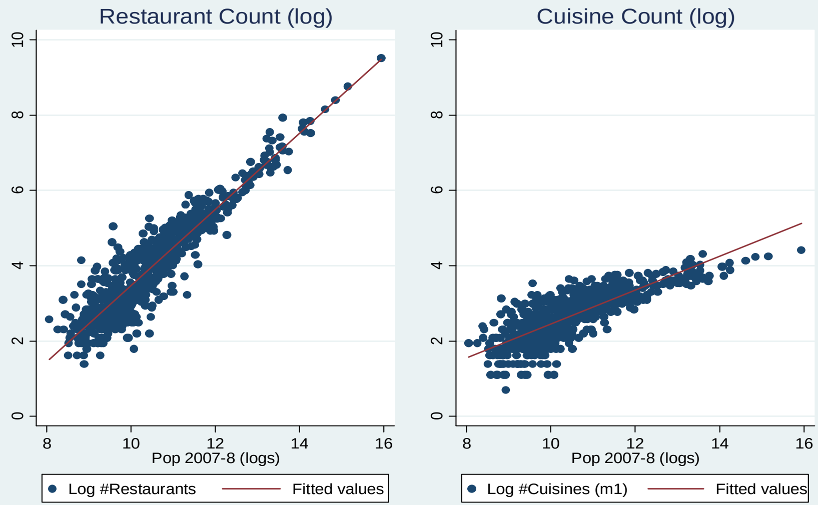
Many models characterize variety as:

1. Symmetric: representative consumer views all varieties as equal (Dixit and Stiglitz, 1979)
2. Unique: each firm is modeled as one variety (# firms= # varieties)

In the context of a consumption amenity I characterize variety as:

1. Asymmetric: some varieties are preferred to others, labels are important
2. Non-unique: classes or categories (ex: clothing styles, music tastes, cuisines), multiple firms compete within the same class

Population, number of restaurants, number cuisines



Slope=1.01, RSq=.86, results for 726 Census Places

Slope=.49, RSq=.67, results for 726 Census Places

Idea and empirical approach

Idea: For industries characterized by significant transportation costs, heterogeneous tastes, and a fixed cost of production, the ability of cities to aggregate niche groups of consumers in a small space will lead to greater variety.

Industry of study: restaurants

- Important consumption amenity of cities
- Cuisines are an easily measured and fairly uncontroversial form of product differentiation
- Transportation costs are important
- Extensive information on industry firms

Key findings

Restaurants exhibit a pattern of cuisines across cities consistent with a model of cuisine-specific entry thresholds that depend upon population and land

- A one std. dev. increase in log population leads to a 57% increase in cuisine count for large cities and a 155% increase for small cities
- Decreasing log land area by one std. dev. increases cuisines by 10% for large cities but has little effect for small cities
- The specific cuisines found in each city follow a hierarchy closely related to population and land–big, dense cities have all varieties found in small, sparsely populated cities but also many varieties not found in the smaller cities

Literature on product variety and cities

Market size and differentiation

1. New Economic Geography models with CES and increasing returns (ex: Krugman 1980)
2. Competition and efficiency: Syverson (2004), Campbell and Hopenhayn (2005)
3. Vertical differentiation: Berry and Waldfogel (2010)
4. Handbury and Weinstein (2012)

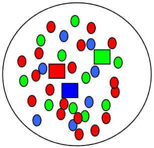
Horizontal differentiation in restaurant industry

1. Waldfogel (2008): local preferences
2. Mazzolari and Neumark (2011): local preferences and local skills

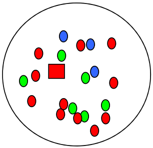
This paper focuses on differentiation (not efficiency) with local preferences but tries to show how general features of cities affect entry.

Main argument: illustrative figure

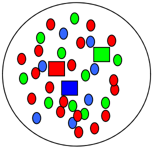
Population=N, 3 Firm Types



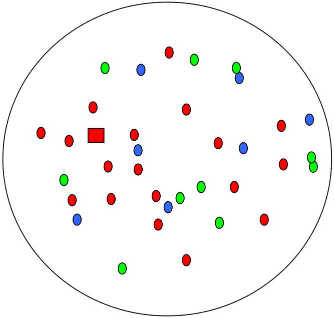
Population=N/2, 1 Firm Type



Population=N, 3 Firm Types

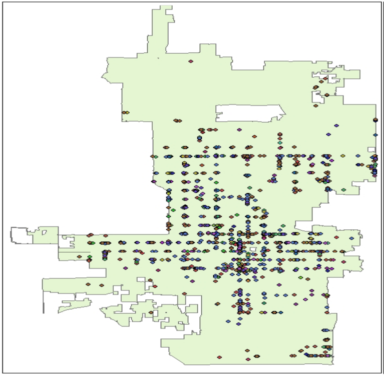


Population=N, 1 Firm Type

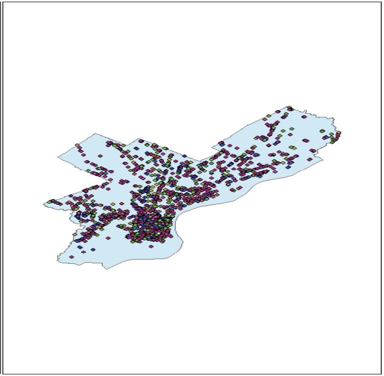


Main argument: Phoenix vs Philly

Phoenix, AZ
Pop: 1.3m
Land: 475 sq mi
Income: $41k
% Coll Educ: 32%
Ethnic HHI: .67
Count Restaurants: 1,865
Count Cuisines: 49



Philadelphia, PA
Pop: 1.5m
Land: 135 sq mi
Income: $31k
% Coll Educ: 24%
Ethnic HHI: .83
Count Restaurants: 2,555
Count Cuisines: 59



Population, land area, and entry

Focus of model: How do population and land area affect the *minimum* conditions for entry of the first firm?

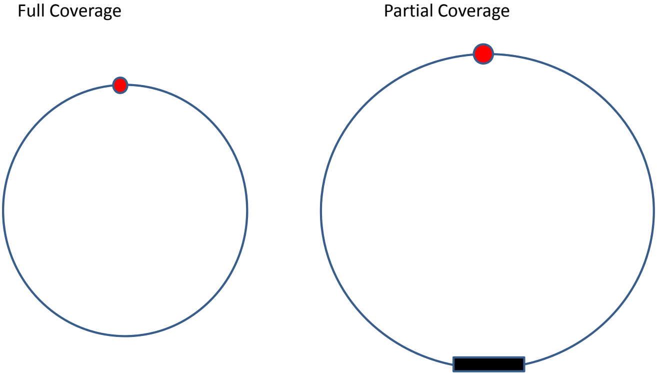
Monopolistic Competition with Reserve Good (Salop, 1979)

- Consumers choose between a firm's product and a reserve good
- Consumers are distributed uniformly around perimeter of a circle; positive transportation cost
- Firms have constant marginal cost and a fixed cost
- Free entry: one firm will enter and make zero profit

Two cases in coverage of market

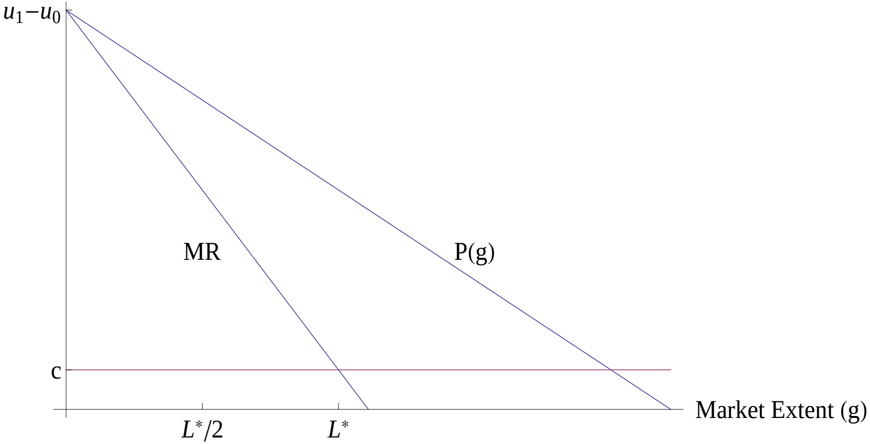
Price determines location of indifferent consumer

Define geographic market extent (g) as distance to indifferent consumer on both sides of firm



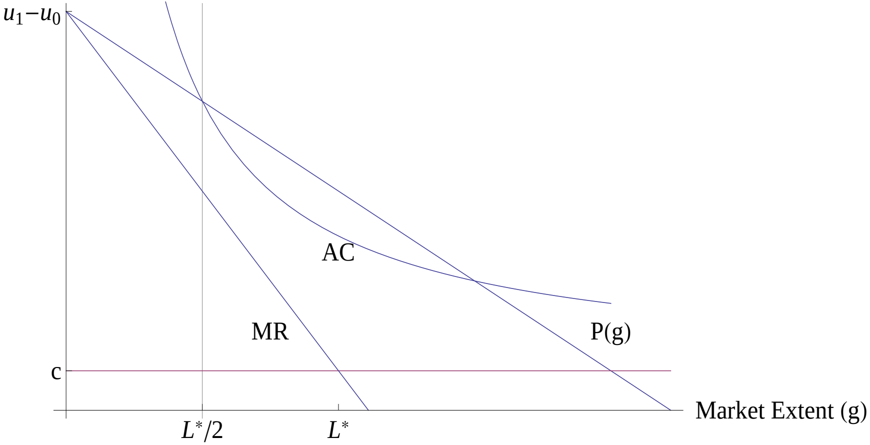
Monopolist chooses market extent to maximize profit

Monopoly profit: $\Pi = D(p(g) - c)g - F = 0$
Small land area constrains monopolist



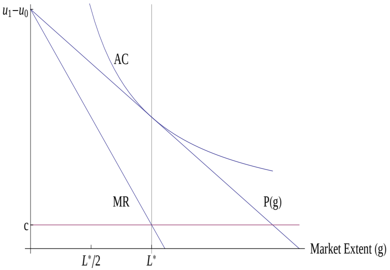
Zero profit condition

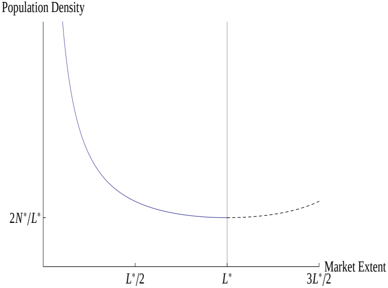
$$\Pi = D(p(g) - c)g - F = 0; p(g) = c + \frac{F}{Dg}$$



Required density for zero profit

For every value of land L there is population density such that profit is zero





Minimum conditions for entry

What is the minimum population for each value of land that would allow entry?

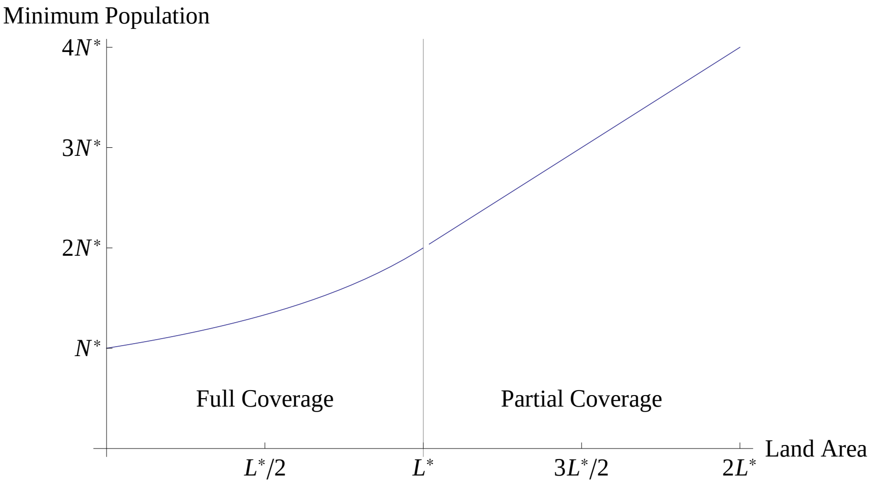
No land: consumers pay entire surplus (over reserve good), minimum population is N^*

Land introduces transportation cost, two cases:

1. Full coverage: firm captures the entire market
2. Partial coverage: firm chooses profit-maximizing market extent L^*; not all consumers purchase good (gaps)

Critical value of land L^* determines which case

Entry frontier in land-population space



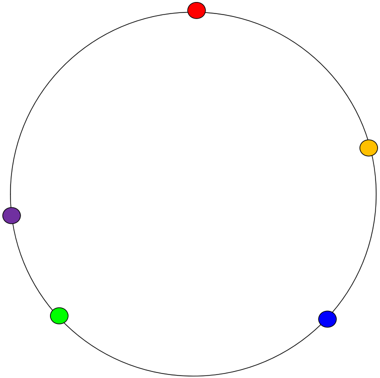
multiple firms

Adding multiple types

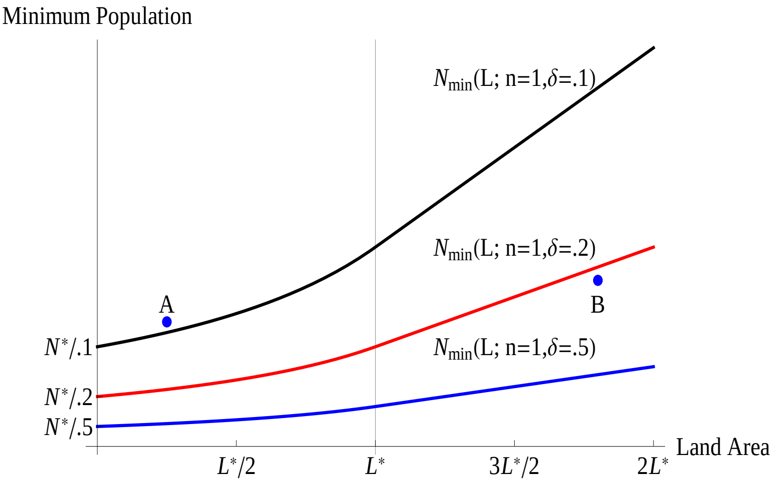
T types of consumers; each consumer of type t demands one unit of type t good

$\Rightarrow$ there is no competition between firms of different types

Comparing across cities (in model); assume fraction δ_t consumers are t type



Multiple types in land-population space



equations

Testable implications of model

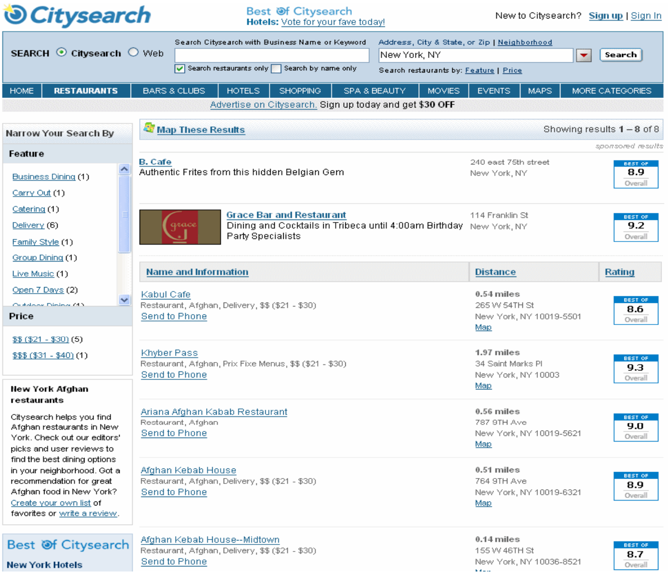
1. Holding land constant, more populous markets will have more types
2. Holding population constant, smaller geographic markets will have more types
3. There will be a hierarchical relationship between the number of types and the composition of those types
4. This hierarchy will be associated with thresholds in population and land; rarer types will be found in bigger, denser markets

Description of data

Collected data from website citysearch.com using software and custom programming in Spring 2007 and Summer 2008

- Restaurants collected for metro areas of 88 of 100 largest US cities, over 300,000 restaurants
- Each restaurant assigned a unique cuisine type (ex: restaurant cannot be pizza and Italian)
- Detailed address information allowed precise placement on map, assigned every restaurant to Census Place
- Matched count of restaurants in every Census Place to count from Economic Census 2007. Kept Census Places with $.7 < \text{match ratio} < 1.1$, leaving 726 places
- Count of restaurants [4,13644], cuisines1 [2,82], cuisines2 [2,277]

data summary table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Feature
Business Dining (1)
Carry Out (1)
Catering (1)
Delivery (6)
Family Style (1)
Group Dining (1)
Live Music (1)
Open 7 Days (2)
Price
$$ ($21 - $30) (5)
$$$ ($31 - $40) (1)
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sponsored results
B. Cafe
Authentic Frites from this hidden Belgian Gem
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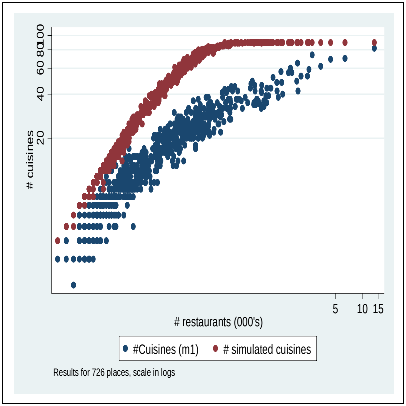
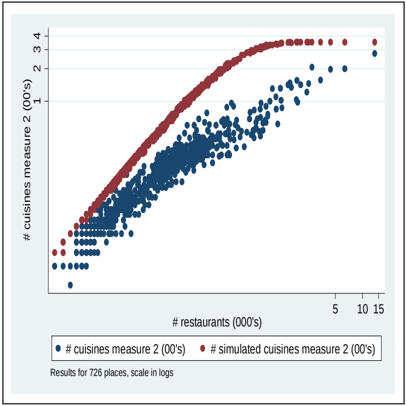
New York, NY
BEST OF 8.9 Overall
Grace Bar and Restaurant
Dining and Cocktails in Tribeca until 4:00am Birthday Party Specialists
114 Franklin St
New York, NY
BEST OF 9.2 Overall
Name and Information
Distance
Rating
Kabul Cafe
Restaurant, Afghan, Delivery, $$ ($21 - $30)
Send to Phone
0.54 miles
265 W 54TH St
New York, NY 10019-5501
Map
BEST OF 8.6 Overall
Khyber Pass
Restaurant, Afghan, Prix Fixe Menus, $$ ($21 - $30)
Send to Phone
1.97 miles
34 Saint Marks Pl
New York, NY 10003
Map
BEST OF 9.3 Overall
Ariana Afghan Kabab Restaurant
Restaurant, Afghan
Send to Phone
0.56 miles
787 9TH Ave
New York, NY 10019-5621
Map
BEST OF 9.0 Overall
Afghan Kebab House
Restaurant, Afghan, Delivery, $$ ($21 - $30)
Send to Phone
0.51 miles
764 9TH Ave
New York, NY 10019-6321
Map
BEST OF 8.9 Overall
Afghan Kebab House--Midtown
Restaurant, Afghan, Delivery, $$ ($21 - $30)
Send to Phone
0.14 miles
155 W 46TH St
New York, NY 10036-8521
BEST OF 8.7 Overall

Number of Cuisines vs. Number of Restaurants

Cuisine Measure 1



Cuisine Measure 2

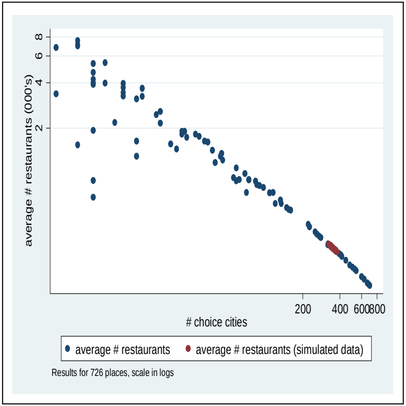


Simulation: n_m draws from uniform multinomial over cuisines, where n_m is the number of restaurants in city m

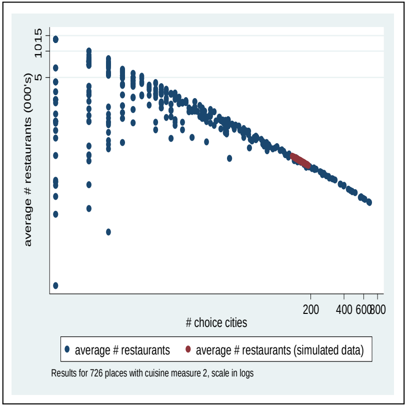
Number Average Size Rule

Average Number of Restaurants in Cities with a Given Cuisine (Mori, Nishikimi, Smith 2008)

Cuisine Measure 1

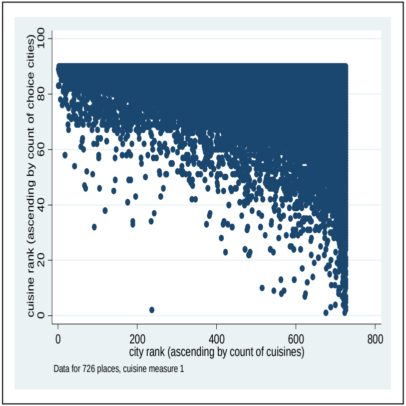


Cuisine Measure 2

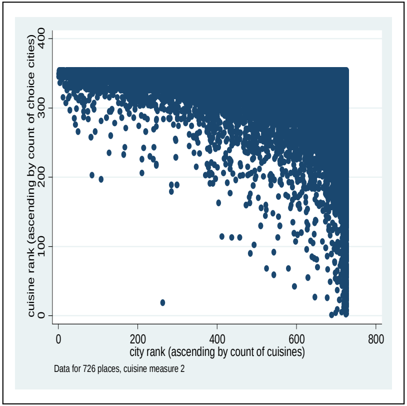


Hierarchy Diagrams (MNS 2008)

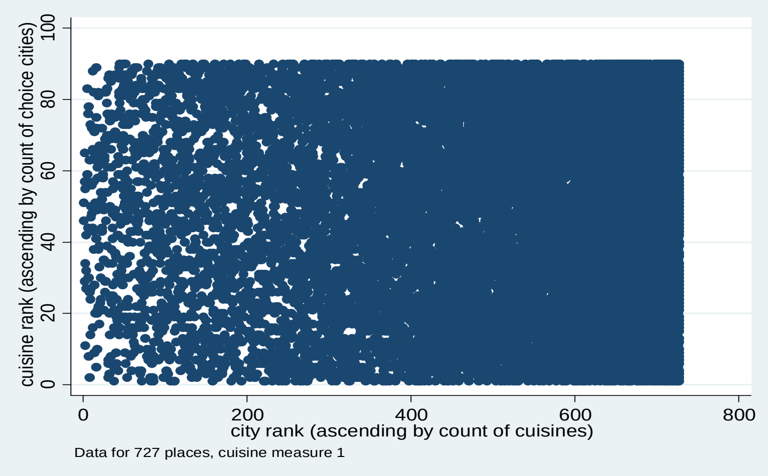
Cuisine Measure 1



Cuisine Measure 2



Hierarchy picture from random assignment



▸ testing procedure

Looking at population thresholds

$$C_{mv} = \begin{cases} 1 & \text{if } N_m - \alpha_v * L_m \geq \frac{N^*}{\delta_v} \\ 0 & \text{o/w} \end{cases}$$

$$Pr(C_{mv} = 1) = Pr(\Pi^*_{mv} > 0)$$
$$\Pi^*_{mv} = \gamma_{1v} N_m + \gamma_{2v} L_m + \eta_v + \epsilon_{mv}$$

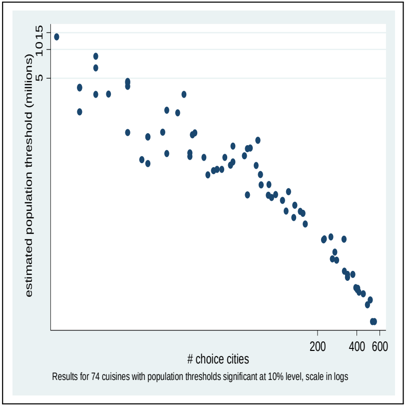
- C_{mv}: binary indicator for variety (cuisine) v in market m
- δ_v percent of people who like variety v
- η_v: cuisine fixed effects (constant)

Run separate regressions for each cuisine

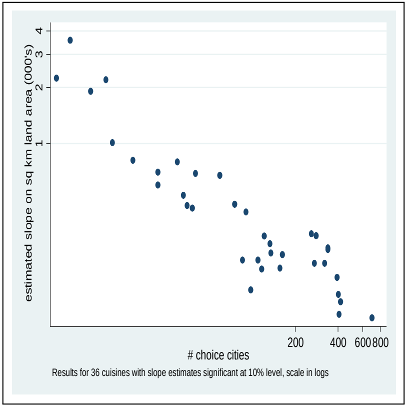
- Population intercept should be higher for rarer cuisines
- Slope of frontier should be higher for rarer cuisines

Intercept and slope estimates

Population Intercept Estimates



Slope on Land Area Estimates



Outline of empirical work

Model predictions:

- Population increases # cuisines, land decreases # cuisines
- Hierarchy related to thresholds in population and land

Testing

1. Cross city regressions on number of cuisines
2. Cuisine level regressions (pooled)
3. Counterfactual simulation
4. Spatial clustering of ethnic populations

Estimating variety across cities

$$ln(\#Cuisines_m) = \gamma_0 + \gamma_1 ln(N_m) + \gamma_2 ln(L_m) + X_m\prime\beta + \epsilon_m$$

- N_m: population of city m
- L_m: land area of city m
- X_m: demographic variables as covariates

$$\begin{aligned} Pr(C_{mv} = 1) &= Pr(\Pi^*_{mv} > 0) \\ \Pi^*_{mv} &= \gamma_1 N_m + \gamma_2 L_m + X_m\prime\beta + \eta_v + \epsilon_{mv} \end{aligned}$$

Predict γ_1 to be positive and γ_2 to negative

Estimate pooled and separately by land quartile

Estimation: number of cuisines

	Log # Cuisines Measure 1				
	All	*Land Qrt4*	*Land Qrt3*	*Land Qrt2*	*Land Qrt1*
Pop 2007-8 (logs)	0.410***	0.332***	0.397***	0.446***	0.457***
	[0.029]	[0.090]	[0.074]	[0.050]	[0.059]
Land sq mtrs (logs)	0.012	0.200*	0.076	0.045	-0.149**
	[0.030]	[0.108]	[0.144]	[0.117]	[0.063]
Average HH Size	-0.479***	-0.513***	-0.456***	-0.301**	-0.393
	[0.080]	[0.151]	[0.159]	[0.132]	[0.237]
Median HH income (000's)	0.003	0.002	0.004	-0.003	-0.002
	[0.002]	[0.005]	[0.003]	[0.005]	[0.005]
Ethnic HHI	-0.543***	-0.888**	-0.812***	-0.462*	0.082
	[0.131]	[0.346]	[0.213]	[0.242]	[0.242]
%Old (>64)	0.292	0.989	1.307	-1.060	-0.086
	[0.562]	[1.462]	[1.131]	[1.207]	[1.431]
%Young (<35)	-0.161	0.774	0.223	-1.277	-0.296
	[0.426]	[1.293]	[0.725]	[1.029]	[0.918]
%College grad	0.619***	0.734	0.661*	0.983**	0.917**
	[0.176]	[0.443]	[0.370]	[0.415]	[0.401]
MSA Fixed Effects	Yes	Yes	Yes	Yes	Yes
Constant	-0.757	-2.607	-1.428	-0.849	1.241
	[0.524]	[1.754]	[2.449]	[2.147]	[1.110]
Observations	703	177	172	175	179
R-squared	0.836	0.697	0.836	0.856	0.910

Likelihood of having a cuisine

	Cuisine Indicator				
Coefficients (marginal effects)	*All*	*Land Qrt4*	*Land Qrt3*	*Land Qrt2*	*Land Qrt1*
Pop 2007-8 (logs)	0.0816***	0.0838***	0.1010***	0.1228***	0.1221***
	[0.0026]	[0.0094]	[0.0067]	[0.0084]	[0.0052]
Land sq mtrs (logs)	-0.0212***	0.0158	-0.0306	-0.0127	-0.0383***
	[0.0028]	[0.013]	[0.0211]	[0.0229]	[0.0065]
Average HH Size	-0.0396***	-0.0470*	-0.0294	-0.0591*	-0.0770**
	[0.0088]	[0.0197]	[0.0166]	[0.0265]	[0.0294]
Median HH income (000's)	0.00	0.00	0.00	0.00	0.00
	[0.0002]	[0.0005]	[0.0005]	[0.0007]	[0.0007]
%Old (>64)	0.0168	0.0283	0.0636	0.1351	-0.0206
	[0.0721]	[0.1452]	[0.1583]	[0.21]	[0.2637]
%Young (<35)	-0.0377	0.0483	-0.1088	0.0395	-0.0236
	[0.0551]	[0.1419]	[0.1085]	[0.1735]	[0.1711]
%College grad	0.1653***	0.1886***	0.1742***	0.2682***	0.2766***
	[0.0213]	[0.0526]	[0.0451]	[0.0633]	[0.0664]
%Corresponding Ethnicity	0.1919***	0.2053***	0.2337***	0.2328***	0.3941***
	[0.0198]	[0.0464]	[0.0447]	[0.0436]	[0.0825]
Cuisine Fixed Effects	YES	YES	YES	YES	YES
Observations	42834	6697	7462	7240	10738
Number of cuisines	59	37	41	40	59
Pseudo R-squared	0.62	0.48	0.55	0.58	0.65
Clustered standard errors in brackets	726 clusters	181 clusters	182 clusters	181 clusters	182 clusters
*** p<0.01, ** p<0.05, * p<0.1					

Counterfactual Simulation

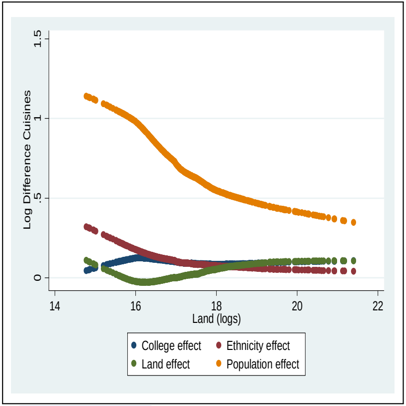
$$Pr(C_{mv} = 1) = Pr(\Pi^*_{mv} > 0)$$
$$\Pi^*_{mv} = \gamma_{1v} N_m + \gamma_{2v} L_m + X_{m'} \beta_v + \eta_v + \epsilon_{mv}$$

Steps

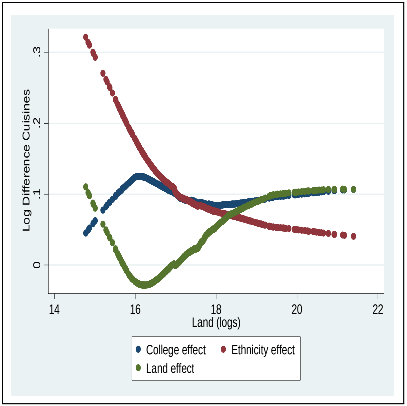
1. Estimate cuisine-specific logits (86 separate regressions) with full set of covariates (including ethnicity, percent college, average HH size)
2. Predict cuisines in each city, denote base case
3. Increase each covariate by one std. dev. (decrease land)
4. Use cuisine-specific logits to re-predict cuisines in each city, compare to base case
5. Show smoothed results of each effect against land area

Simulation results

All effects



No Population



Simulation Table

Ethnicity and space

Alternative supply-side story: big dense cities have greater variety of skilled producers

- Arguably less important explanation: much harder to move demand
- Cannot be ruled out without dataset on restaurant producers

Will show evidence more suggestive of critical mass of demand:

1. Show city-level spatial concentration of an ethnic group predicts presence of ethnic restaurant
2. Show that ethnic population size predicts location of ethnic restaurant at *tract* level

▸ Formula

Spatial clustering of ethnic populations

Panel A: Census place level			*Panel B: Census tract level*		
Coefficients (marginal effects)	Cuisine Indicator	Cuisine Indicator	Coefficients (OLS)	Cuisine Indicator	Cuisine Indicator
Pop 2007-8 (logs)	0.1327***	0.1196***	Corresponding ethnic population	0.024***	
	[0.0057]	[0.006]	(000's)	[0.002]	
Land sq mtrs (logs)	-0.0422***	-0.0367***	Remaining population (000's)	0.002***	
	[0.0071]	[0.0072]		[0.000]	
Average HH Size	-0.0821**	-0.0800**	Average HH Size	-0.028***	-0.027***
	[0.0317]	[0.031]		[0.003]	[0.003]
Median HH income (000's)	-0.0007	-0.0007	Median HH income (000's)	-0.000**	-0.000**
	[0.0008]	[0.0008]		[0.000]	[0.000]
%Old (>64)	-0.0701	-0.067	%Old (>64)	0.034***	0.038***
	[0.2813]	[0.2817]		[0.006]	[0.006]
%Young (<35)	-0.047	-0.0468	%Young (<35)	0.028***	0.035***
	[0.1865]	[0.186]		[0.006]	[0.006]
%College grad	0.3032***	0.3073***	%College grad	-0.025*	-0.023
	[0.072]	[0.0711]		[0.015]	[0.016]
%Corresponding Ethnicity	0.4277***	0.3723***	%Corresponding Ethnicity		0.292***
	[0.0893]	[0.0939]			[0.015]
Moran's I		0.1358***	Constant	0.053***	0.051***
		[0.0222]		[0.008]	[0.008]
Cuisine Fixed Effects	YES	YES	Cuisine Fixed Effects	YES	YES
			Census Place Fixed Effects	YES	YES
Observations	9790	9790	Observations	959753	959753
Pseudo R-squared	0.634	0.639	R-squared	0.236	0.237
Clustered standard errors in brackets (182 clusters)			Robust standard errors in brackets (726 clusters)		
*** p<0.01, ** p<0.05, * p<0.1			*** p<0.01, ** p<0.05, * p<0.1		

Summary of findings

Both population and population density affect variety of non-tradable consumer goods in cities

- variety rises very slowly with population; only large increases in population increase variety count
- partial effect of land area alone is persistent for geographically large cities but magnitude is small
- cuisine diversity is higher in big dense cities due to additional cuisines
- bigger denser cities are more likely to have any type; rarer types are found in cities with greater populations and smaller land areas

Interpretation

City structure–geographic distribution of population–may directly increase consumption good diversity by aggregating heterogeneous preferences in space

Hierarchical relationship is consistent with a model of entry thresholds and increasingly rare tastes

Urban policies (ex: zoning) encouraging density may lead to greater variety and provision of varieties appealing to minority tastes

End of main slides

Thank you!

Data Summary Table

	Land Qrt 4 (n=181)			Land Qrt 3 (n=182)			Land Qrt 2 (n=181)			Land Qrt 1 (n=182)		
	Mean	Std. Dev.	[Min, Max]	Mean	Std. Dev.	[Min, Max]	Mean	Std. Dev.	[Min, Max]	Mean	Std. Dev.	[Min, Max]
# Restaurants	27.1	29.4	[4, 192]	51.8	51.9	[5, 359]	91.2	72.0	[6, 380]	531.3	1241.9	[8, 13664]
# Cuisines (m1)	10.4	6.7	[2, 38]	14.5	7.8	[3, 43]	18.7	8.1	[3, 45]	29.1	14.1	[3, 82]
# Cuisines (m2)	12.6	11.0	[2, 78]	18.5	13.6	[3, 96]	25.2	14.5	[3, 90]	49.5	40.7	[4, 277]
Population 2007-08 (thousands)	16.12	10.97	[3.14, 75.7]	29.30	20.95	[4.6, 107.05]	53.78	35.87	[6.11, 239.18]	331.38	750.08	[7.15, 8328.5]
Land Area (sq km)	9.79	3.16	[2.61, 14.93]	21.73	4.28	[14.95, 29.99]	43.95	9.10	[30.12, 61.31]	229.89	296.76	[61.54, 1962.37]
Density (Pop per sq km)	1,766	1,308	[326, 12143]	1,342	935	[243, 6429]	1,233	802	[175, 6191]	1,315	1,192	[55, 10601]
MSA Population 2000 (millions)	5.39	5.06	[0.30, 21.20]	5.64	5.02	[0.30, 21.20]	5.46	5.06	[0.15, 21.20]	4.52	4.49	[0.15, 21.20]
Average HH Size	2.59	0.45	[1.71, 4.37]	2.59	0.32	[1.82, 3.59]	2.61	0.29	[1.98, 3.65]	2.62	0.31	[2.02, 4.12]
Median HH Income (thousands)	$50.0	$17.6	[$17.7, $134.3]	$50.9	$16.6	[$24.2, $146.5]	$56.2	$19.3	[$26.8, $139.9]	$49.1	$15.4	[$24.5, $111.8]
Ethnic HHI	0.79	0.19	[0.26, 0.99]	0.80	0.19	[0.25, 0.99]	0.78	0.18	[0.17, 1]	0.76	0.15	[0.23, 0.97]
%Young (<35yrs)	14%	6%	[4%, 43%]	13%	5%	[3%, 37%]	12%	5%	[3%, 30%]	10%	4%	[3%, 34%]
%Old (>64yrs)	48%	8%	[21%, 69%]	49%	7%	[27%, 81%]	49%	6%	[33%, 68%]	52%	6%	[28%, 66%]
%College (completed for 25yrs+)	33%	17%	[4%, 81%]	36%	15%	[10%, 75%]	39%	16%	[11%, 78%]	36%	13%	[7%, 71%]

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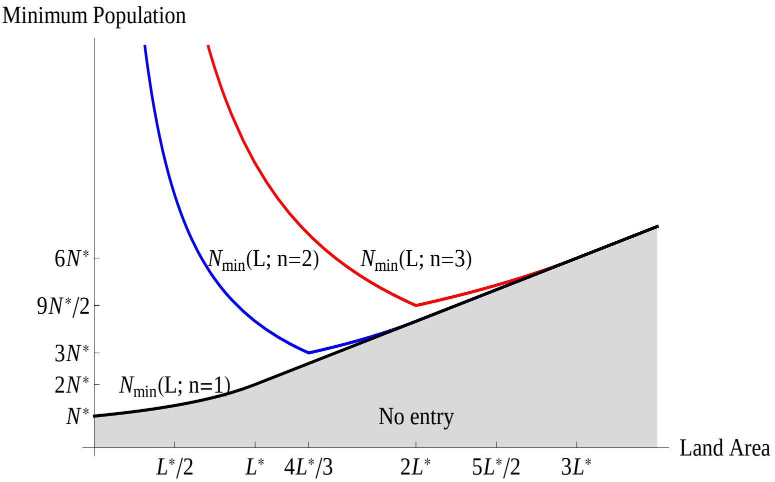
Minimum market conditions: multiple firms



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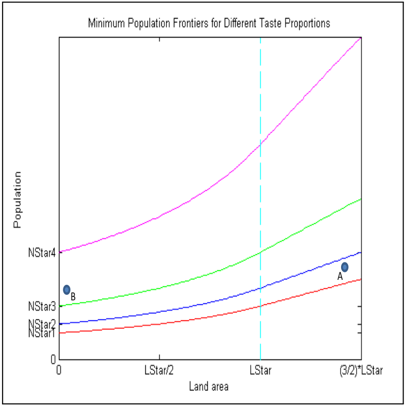
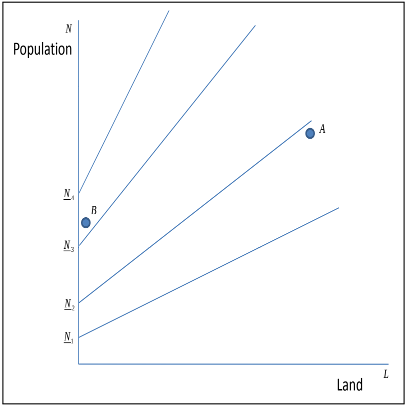
Likelihood of having a cuisine: simpler specification

Model Specification



Probit Specification



Testing hierarchy: random labeling hypothesis

H_0: cuisine labels are drawn uniformly from set of cuisines

Testing procedure (Mori, Nishikimi, Smith 2008)

- for each city randomly draw cuisine labels from total set
- calculate hierarchy share: count of events where cuisine is found in all more diverse cities
- run simulation 10,000 times to generate p-value

	Cuisine Measure 1	Cuisine Measure 2
726 Cities	23%***	15%***

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Simulation results

Land Quartile	Cuisine Type	Baseline Count	ΔPopulation Change in cuisine count		ΔLand Change in cuisine count		ΔCollege Change in cuisine count		ΔEthnic Change in cuisine count	
1	Non-ethnic	15.60	6.03	39%	0.91	6%	1.10	7%	0.00	0%
	Ethnic	12.69	10.18	80%	1.84	15%	1.77	14%	1.67	13%
	Total	***28.30***	***16.20***	***57%***	***2.75***	***10%***	***2.87***	***10%***	***1.67***	***6%***
2	Non-ethnic	10.82	6.04	56%	0.56	5%	0.70	6%	0.00	0%
	Ethnic	7.18	7.05	98%	0.75	10%	0.99	14%	1.56	22%
	Total	***18.01***	***13.09***	***73%***	***1.31***	***7%***	***1.69***	***9%***	***1.56***	***9%***
3	Non-ethnic	7.41	6.48	87%	0.06	1%	0.62	8%	0.00	0%
	Ethnic	5.34	5.41	101%	0.35	7%	0.72	13%	1.25	23%
	Total	***12.75***	***11.90***	***93%***	***0.41***	***3%***	***1.34***	***10%***	***1.25***	***10%***
4	Non-ethnic	4.49	6.93	154%	0.22	5%	0.57	13%	0.00	0%
	Ethnic	3.52	4.68	133%	0.00	0%	0.59	17%	1.24	35%
	Total	***8.01***	***11.61***	***145%***	***0.22***	***3%***	***1.16***	***14%***	***1.24***	***15%***

Table shows average change in count of cuisines resulting from a one standard deviation* ***decrease* *in log land area and a one standard deviation* ***increase*** *in every other variable. Percent change is calculated as percent of baseline count.*

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Moran's I

$$I = \frac{N}{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}} \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(X_i - \bar{X})(X_j - \bar{X})}{\sum_{i=1}^{N}(X_i - \bar{X})^2}$$

Ethnicity and Space

Equations for minimum population frontier

$$N_{min}(L;\delta_v) = \begin{cases} \frac{1}{\delta_v} * \frac{2N^* L^*}{2L^* - L} & \text{if } L \leq L^*, \text{"full coverage"} \\ \frac{1}{\delta_v} * \frac{2N^* L}{L^*} & \text{if } L^* < L, \text{"partial coverage"} \end{cases} \quad (1)$$

$$\frac{\partial N_{min}(L;\delta_v)}{\partial L} = \begin{cases} \frac{\alpha_v L^*}{(2L^* - L)^2} & \text{if } L \leq L^*, \text{"full coverage"} \\ \frac{2N^*}{\delta_v L^*} = \alpha_v & \text{if } L^* < L, \text{"partial coverage"} \end{cases} \quad (2)$$

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